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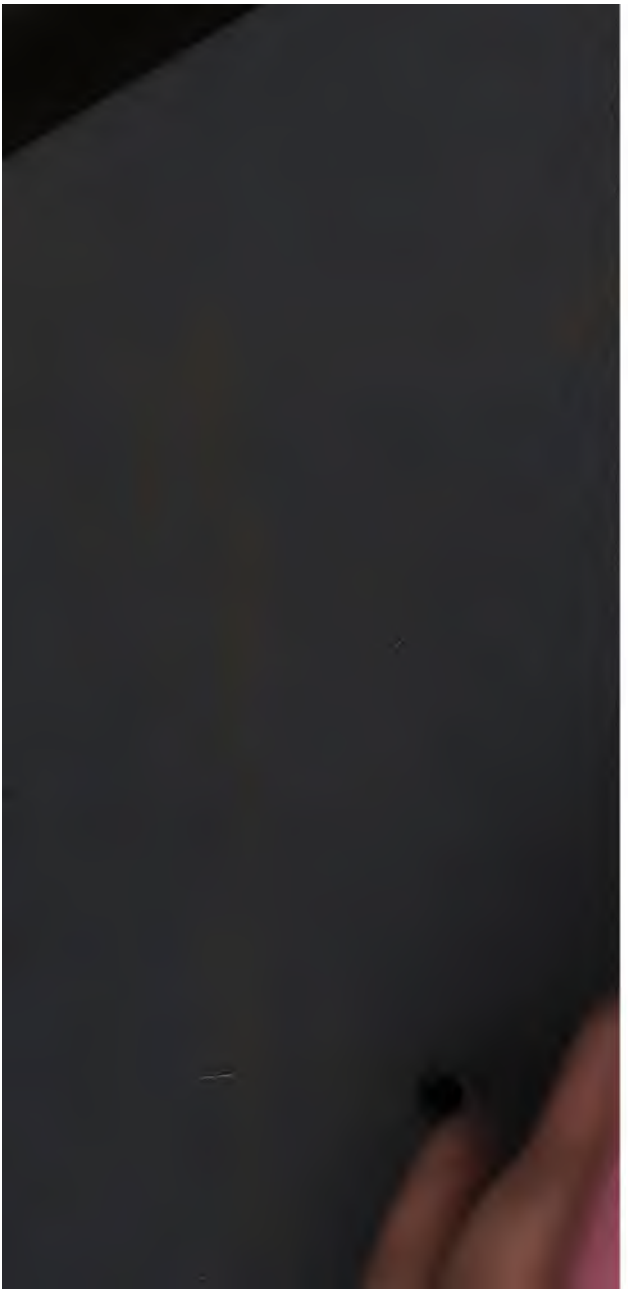

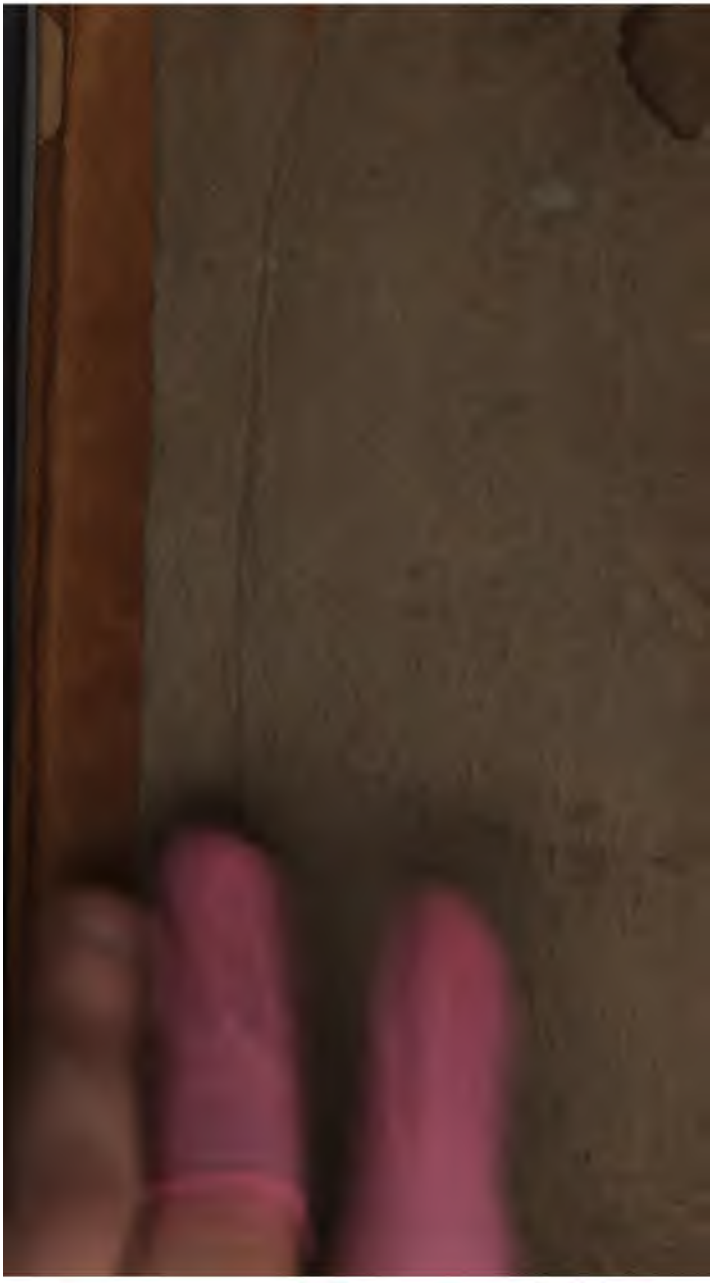

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PRACTICAL ARITHMETIC:

OR

A COMPLETE

EXERCISE-BOOK,

FOR THE

USE OF SCHOOLS.

By NATHAN T. HILTON.

EXETER:

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and by CHARLES PEIRCE, Portsmouth.

1807.

PREFACE.

BOOKS upon the subject of arithmetic have been, of late, very numerous; but whether the subject has been wholly exhausted or not, I leave to be considered. To me it appeared, that there was room for improvement both in matter and form; and whether I have affected it, in any degree, must also remain to be considered. The improvement to which I advert, and would be understood chiefly to mean, is in arithmetic as it relates to schools, in which should be comprised all the rules upon that science, expressed in a concise, comprehensive, and methodical manner, with the first example under each, wrought out at large according to the rule, and the rest in question and answer. This is the plan which I have prosecuted in the following work.

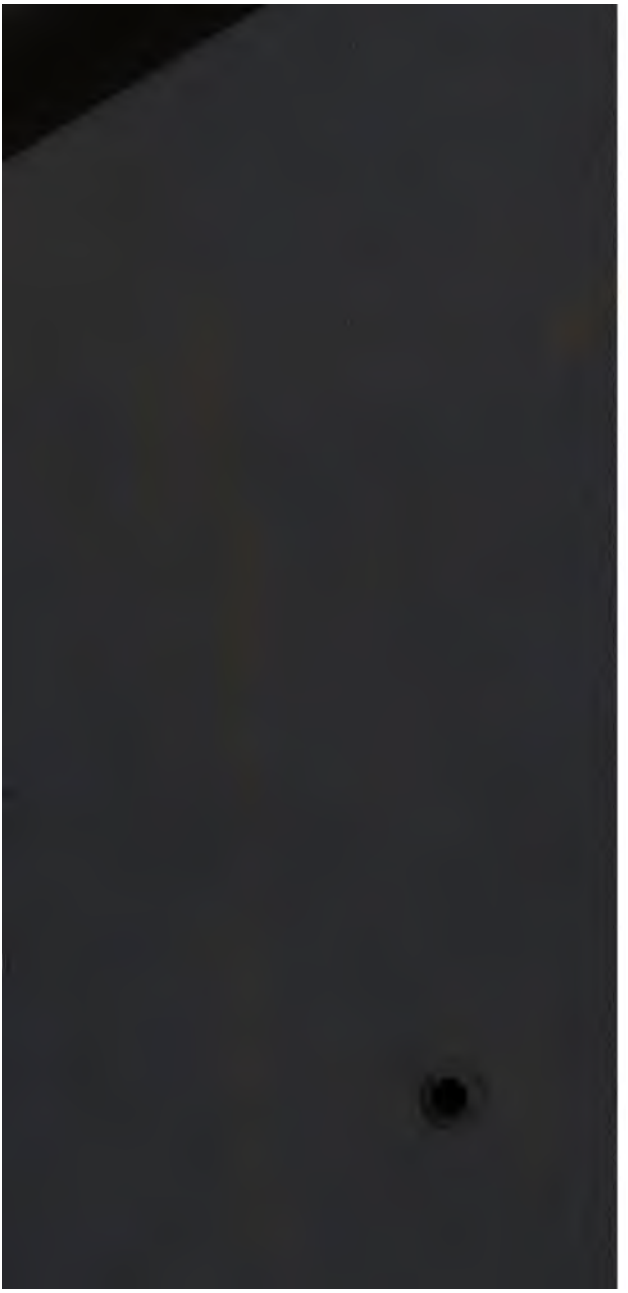
The method, perhaps, may not suit some, though it appeared to me to be the best; however, the rules may be transcribed in any other arrangement at pleasure.

The work is large, comprehended in a small compass, and almost entirely new.

Being conscious that the following compendium will stand or fall according to its merit or demerit, I therefore, without any recommendation, submit it to the examination of an impartial public.

THE AUTHOR.

Newmarket, N. H. Aug. 7
the 25th, 1806.



Duodecimals.
Proportion in general.
Simple Proportion, or Rule of Three.
Compound Proportion, or Double Rule of Three.
Conjoined Proportion.
Practice.
Tare and Trett.
Barter.
Loss and Gain.
Fellowship.
Allegation.
Involution.
Evolution.
Extraction of the Square Root.
————Cube Root.
————Roots of powers in general.
Arithmetical Progression.
Geometrical Progression.
Simple Interest.
———by Decimals.
Discount.
———by Decimals.
Commission.
Brokerage.
Buying and selling of Stocks.
Insurance.
Compound Interest.
———by Decimals.
Discount by Compound Interest.
Annuities.
Single Position.
Double Position.
Permutations and Combinations.
Miscellaneous Questions.

EXPLANATION OF CHARACTERS.

=	The sign of *equality*; as, 10 dimes = 1 dollar, signifies that 10 dimes is equal to 1 dollar.
+	Signifies *plus*, or *addition*; as, 8+2=10.
—	Signifies *minus*, or *subtraction*; as, 10—2=8.
×	The sign of *multiplication*; as, 6×5=30, *i.e.* 6 multiplied by 5 is equal to 30. Read 6 into 5 is equal to 30.
÷ or 5)30(6	Signifies *division*; as, 30 ÷ 5 = 6, or 5)30(6.
$\frac{30}{5}$	Division may likewise be denoted by placing the dividend above a line, and the divisor under it; thus, $\frac{30}{5} = 30 \div 5 = 6$.
∴	Signifies *arithmetical proportion*; as, 2 . . 4 : : 6 . . 8; here the meaning is, that 4 — 2 = 8 — 6 = 2.
: : : :	Signifies *geometrical proportion*; thus, 2 : 4 : : 8 : 16, which is to be read, as 2 is to 4 so is 8 to 16.
∴	Signifies *therefore*.
$\square^2$	Signifies the *second power*, or *square*.

$\overline{\quad}|^{3}$ Signifies *the third power*, or *cube*.

$\overline{\quad}|^{m}$ Signifies *any power*.

$\sqrt{}$ or $\overline{\quad}|^{\frac{1}{2}}$ Signifies *the square root*.

$\sqrt[3]{}$ or $\overline{\quad}|^{\frac{1}{3}}$ Signifies *the cube root*.

$\sqrt[m]{}$ or $\overline{\quad}|^{\frac{1}{m}}$ Signifies *any root*.

$\overline{\quad}|^{\frac{n}{m}}$ Signifies *any root of any power*.

Note. The number or letter, belonging to the above signs of powers and roots is called the *index* or *exponent*.

$30-\overline{8+6}=16$ A line, or vinculum, drawn over several numbers, signifies, that the numbers under it are to be considered jointly; thus, $30-\overline{8+6}=16$; but without the vinculum, $30-8+6=28$.

abc When several letters are placed together, they are supposed to be multiplied into one another; thus, $abc = a \times b \times c =$ the product of a, b, and c.

ARITHMETIC.

ARITHMETIC is the art or science of computing by numbers, and has five principal rules for its operations, viz....*Notation* or *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

NUMERATION.

Numeration is the expression of number by words or characters. The characters by which all numbers may be denoted, are these ten, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Besides this simple value of figures, they have, each, a local value, according to the following *Example*.

&c.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
&c.	9	8	7	6	5	4	3	2	1
		9	8	7	6	5	4	3	2
			9	8	7	6	5	4	3
				9	8	7	6	5	4
					9	8	7	6	5
						9	8	7	6
							9	8	7
								9	8
									9

*To express in words any number proposed in figures.**

Rule. To the simple value of each figure, join the name of its place, beginning at the left hand and reading towards the right.

EXAMPLE.

Write down in words the following number, *viz.* 9099 09909.

To write numbers.

Rule. Write down the figures as their values are expressed, and supply any deficiency in the order with cyphers.

Example. Write down in figures the following number, *viz.* Four hundred and thirty thousand, one hundred and twenty three millions four hundred and fifty six thousand, seven hundred and eighty nine.

SIMPLE ADDITION.

Simple Addition is the collecting of several integers or whole numbers of the same kind into one sum.

RULE.

Place the numbers under one another, so that units may stand under units, tens under tens, &c. and draw a line under them. Add up the figures in the first column, or row of units, and place the units under it, and carry the tens to the next column: proceed in the same manner through every row, or column, and set down the whole amount of the last column.

PROOF.

Cut off the uppermost number by drawing a line under it: add all the rest together, and set their sum under the number to be proved: then, add the last found num-

* For Notation by numerical letters, see the Young Lady and Gentleman's Spelling Book. Chap. 9.

ber and the uppermost together, and if the sum be equal to that found by the first addition, the work is right.

EXAMPLES.

1.	2.	3.	4.
123456	123456	3	123456
789123	78912	12	789012
456789	3456	789	345678
123456	789	3456	901234
789123	12	78912	567890
456789	3	123456	123456
Sums.			
Proof.			

5. Add 3400645, 1231231, 9000870, 8456009, 147 6000, 4500000, 1234567, and 8900000 together.

Ans. 38199322.

6. Add 24096735, 1220400, 36675, 400000, 878, 87654321, 987654, and 12345678 together.

Ans. 126742341.

SIMPLE SUBTRACTION.

Simple Subtraction is the finding of the difference between any two numbers of one kind or denomination, by taking the less from the greater.

RULE.

Place the less number under the greater, so that units may stand under units, tens under tens, &c. then, draw a line under them: begin with the units, and subtract the lower from the upper figure, and set down the remainder; but if the lower figure be greater than the upper, borrow ten, and add it to the upper figure, from which

take the lower, setting down the difference, ... you must add one to the ten's place of the less numb... er for that which you borrowed; and thus proceed t... rough the whole.

PROOF.

Add the remainder to the less number, and if the sum be equal to the greater, the work is right.

EXAMPLES.

	1.	2.	3.
From	675496	765432	987654321
Take	503263	123456	123456789
Rem.	172233		
Proof	675496		

4. From 4360000 take 3001283. *Ans.* 1358717.
5. From 56780946 take 20990900. *Ans.* 35790046.
6. From 100000000 take 12345679.
Ans. 87654321.

SIMPLE MULTIPLICATION.

Simple Multiplication is a short method of performing the work of simple addition.

The number to be multiplied is called the Multiplicand.*

The number you multiply by is called the Multiplier.†

The number found, after the work is done, is called the product.‡

*†‡ Both the multiplicand and multiplier are, in general, called Terms, or Factors; and the product is sometimes called the Answer, Fact, or Rectangle.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

CASE 1.

When the multiplier is not more than 12.

RULE.

Place the multiplier under the multiplicand, units under units, &c. then, begin as the table directs; set down the unite figure under units, and carry the tens to the next place, in all respects as in simple addition.

PROOF.

Multiply the multiplier by the multiplicand.

EXAMPLES.

1.	2.	3.
28424	678496	4980456
2	3	4
Products.		

4.	5.	6.
762009	8003243	95167804
5	6	7

7.	8.	9.
5600072	123456789	987654321
8	9	10

10.	11.
8090607054	980090008906703
11	12

CASE 2.

When the multiplier consists of more places than one.

Rule.

Multiply every figure in the multiplicand by each figure in the multiplier, beginning with the units, and place the first figure of each product exactly under its multiplier: then add all the products together, in the order in which they stand, and their sum will be their total product.

Examples.

1.	2.
Multiply 2321	23
by 23	2321
6963	23
4642	46
	69
Product 53383	46
	53383 *Proof.*

2. Multiply 36742 by 47. *Ans.* 1726874.
3. Multiply 560345 by 78. *Ans.* 43706910.
4. Multiply 496123 by 342. *Ans.* 169674066.
5. Multiply 1604382 by 678. *Ans.* 1087770996.
6. Multiply 9204531 by 3624. *Ans.* 33357220344.
7. Multiply 123456 by 54321. *Ans.* 6706253376.
8. Multiply 2463857 by 759243. *Ans.*1870666180251.
9. Multiply 900047021 by 8765932.
Ans. 7889750982888572.
10. Multiply 333004444 by 23423451.
Ans. 7800113276816244.
11. Multiply 987654321 by 123456789.
Ans. 121932631112635269.
12. Multiply 6320900145 by 987654321.
Ans. 6242864340818776545.

CASE 3.

When the multiplier is a composite number, that is, when it is produced by the multiplication of two, or more, numbers in the Table.

RULE.

Multiply the multiplicand by one of those numbers; that product by another, and so on; and the last product will be the total required.

EXAMPLES.

1.

Multiply 176342 by 45.

$9 \times 5 = 45$ $6 \times 5 = 45$.

1587078
5

Product 7935390

2. Multiply 2536478 by 56. *Ans.* 142042768.
3. Multiply 300006 by 63. *Ans.* 18900378.
4. Multiply 4300681 by 72. *Ans.* 309649032.

5. Multiply 56789012 by 81. *Ans.* 4599909972.
6. Multiply 5432432 by 88. *Ans.* 478054016.
7. Multiply 623701 by 108. *Ans.* 67359708.
8. Multiply 74000074 by 121. *Ans.* 8954008954.
9. Multiply 123456789 by 144. *Ans.* 17777777616.
10. Multiply 987654321 by 1440. *Ans.* 1422222222240.

CASE 4.

When there are cyphers on the right hand of, either the multiplicand, or multiplier, or both.

RULE.

Place the factors in such order, without regarding the cyphers, that the figures may stand exactly under one another, and multiply by them only; add the products together, as before directed, and place to the right hand as many cyphers as there are in both the factors.*

EXAMPLES.

1.

456700
38

36536
13701

17354600 *Product.*

* To multiply by 10, 100, 1000, &c. only annex as many cyphers to the multiplicand, as there are cyphers in the multiplier, and the work is done.

EXAMPLE.

Multiply 123456 by 1000.
1000

Ans. 123456000

2. Multiply 64543 by 57000. *Ans.* 3678951000.
3. Multiply 3678000 by 860. *Ans.* 3163080000.
4. Multiply 63210 by 321000. *Ans.* 20290410000.
5. Multiply 1234000 by 23400. *Ans.* 28875600000.
6. Multiply 342500 by 2435000. *Ans.* 833987500000.
7. Multiply 600030000 by 26341000.
Ans. 15805390230000000.
8. Multiply 740051000 by 3216450000.
Ans. 2380337038950000000.
9. Multiply 56780000 by 56780000.
Ans. 3223968400000000.

CASE 5.

When there are cyphers between the figures of the multiplier.

RULE.

Multiply as usual, without regarding the cyphers, and place the first figure of each product exactly under that figure by which you multiply; add them together, and their sum will be their total product.

EXAMPLES.

1.

Multiply 6452 by 2003.

2003

19356
12904

12923356 *Product.*

2. Multiply 5032 by 304. *Ans.* 1529728.
3. Multiply 54009 by 50002. *Ans.* 2700558018.
4. Multiply 312452 by 100023. *Ans.* 31252386396.
5. Multiply 43214321 by 4030002.
Ans. 174153800058642.
6. Multiply 9012345678900 by 3000040050600.
Ans. 27037397986551847352310000.

SIMPLE DIVISION.

Simple Division is a compendious rule of performing several simple subtractions. It consists of four parts, *viz.*

1. The Dividend, or number given to be divided.
2. The Divisor, or number given to divide by.
3. The Quotient, or answer to the question, which shews how often the divisor is contained in the dividend.
4. The Remainder, which is always less than the divisor, and of the same name with the dividend, is very uncertain, as there is sometimes a remainder, and sometimes not.

CASE 1.

RULE. Consider how many times the divisor is contained in the first figures of the dividend; when found, place the figure in the quotient; multiply the divisor by this quotient figure, and place the product under the left hand figures of the dividend; then subtract it therefrom, and to the right hand of the remainder bring down the next figure of the dividend: If, when you have brought down a figure to the remainder, it be still less than the divisor, a cypher must be placed in the quotient, and another figure must be brought down; after which, you must divide as before, till you have brought down every figure of the dividend.

PROOF.

Multiply the divisor and quotient together, to the product add the remainder, if there be any, and the sum will be equal to the dividend, when the work is right.

EXAMPLES.

Dividend.

Divisor. 23)53395(2321 $\frac{12}{23}$. *Quotient.*

46

73	2321—*Quotient.*
69	23×*Divisor.*
49	6963
46	4642
35	53383
23	12+*Rem.*
Remainder. 12	53395 *Proof.*

2. Divide 1726874 by 47. *Ans.* 36742.
3. Divide 43706946 by 78. *Ans.* 560345 $\frac{16}{78}$.
4. Divide 16967406771 by 342. *Ans.* 49612300 $\frac{171}{342}$.
5. Divide 1087771341 by 678. *Ans.* 1604382 $\frac{345}{678}$.
6. Divide 9876543210 by 5009. *Ans.* 1971759 $\frac{2179}{5009}$.
7. Divide 6706255721 by 54321. *Ans.* 123456 $\frac{23451}{54321}$.
8. Divide 69601915210464 by 800016. *Ans.* 87000654.
9. Divide 1870666181451068 by 2463857.
Ans. 759243000 $\frac{12000068}{24638577}$.
10. Divide 7800113281361241 by 23423451.
Ans. 333004111 $\frac{4545000}{2342345}$.
11. Divide 15241578750190521 by 123456789.
Ans. 123456789.
12. Divide 1219326311126352 69 by 987654321.
Ans. 123456789.

CASE 2.

When there are cyphers at the right hand of the divisor.

RULE.

Cut them off; also, the same number of places from the dividend, and work as in Case first: But if the places,

which were cut off from the dividend, be figures, they must be placed at the right hand of the remainder at last.*

EXAMPLES.

1.

31|00)230677|34(6781 $\frac{1114}{1400}$.
201

266
238

287
272

157
136

2134

2. Divide 105777468645 by 306000.
Ans. 345678 $\frac{645}{306000}$.

3. Divide 511108957364 by 4140000.
Ans. 123456 $\frac{1117364}{4140000}$.

4. Divide 97558007000000 by 324500000.
Ans. 300641 $\frac{25}{325}$.

5. Divide 5333326000000 by 1234000000.
Ans. 4321 $\frac{1213}{1234}$.

6. Divide 45350332685000000 by 67420000000.
Ans. 672654.

CASE 3.

To perform division without setting down the multiplication.

---

*In dividing by 10, 100, 1000, 10000, &c. when you cut off as many places from the dividend, as there are cyphers in the divisor, your work is done; those figures, cut off on the right hand, are the remainder, and those on the left, the quotient.

EXAMPLE.

1|000)1234|567 4321|0000 by 1|0000.
Ans. 1234 $\frac{567}{1000}$. *Ans.* 4321.

RULE.

Find the first figure of the quotient as usual, and multiply the divisor by it; subtract each figure of the product, as you produce it, from the dividend; and, if you borrow in subtracting, you must carry one extraordinary to the next multiplication for the one borrowed, and thus proceed, till the division is completed.

EXAMPLES.

23)53383(2321
73
48
23

2. Divide 132279293 by 365. *Ans.* 362409 $\frac{8}{365}$.
3. Divide 1673652552 by 4092. *Ans.* 409006.
4. Divide 9876543210 by 12345. *Ans.* 800014 $\frac{10}{12345}$.

CASE 4.

When the divisor does not exceed 12.

RULE.

Find the number of times the divisor is contained in the first figure, or figures of the dividend, as before; which place in the quotient; then, mentally, multiply the divisor by the figure placed in the quotient, and subtract the product from the left hand figure, or figures, of the dividend: the remainder you must suppose to stand on the left hand of the next figure of the dividend, and to be reckoned with it; then you must find how often you can have the divisor in those figures; but, if there be no remainder, you must find the number of times the divisor is contained in the next figure; and thus go on till you have done.

Examples.

Dividend.
Divisor. 2)4326563

Quotient. 2163281—1 *Remainder*

2. Divide 45674567 by 3. *Ans.* 15224855 $\frac{2}{3}$.
3. Divide 47865065 by 4. *Ans.* 11966266 $\frac{1}{4}$.
4. Divide 8810045 by 5. *Ans.* 1762009.
5. Divide 86000047 by 6. *Ans.* 14333341 $\frac{1}{6}$.
6. Divide 666174628 by 7. *Ans.* 95167804.
7. Divide 7200071797 by 8. *Ans.* 900008974 $\frac{5}{8}$.
8. Divide 1111111101 by 9. *Ans.* 123456789.
9. Divide 88996677594 by 11. *Ans.* 8090607054.
10. Divide 11761080106880436 by 12.
Ans. 980090008906703.

CASE 5.

When the divisor is a composite number, that is, when it is the product of any two, or more numbers.

Rule.

Divide the given dividend by one of those numbers; the quotient, thence arising, by another, and so on; and the last quotient will be the answer.*

---

* Since, as it sometimes happens, that there is a number of remainders, resulting from an operation by this Rule, and as the quotient is incomplete without the true remainder; I shall here give a Rule for finding it.

Rule.—Multiply the last divisor but one, by the last remainder, to the product add the next preceding remainder; multiply this sum, by the next preceding divisor, and to the product add the next preceding remainder; and so on, till you have gone through all the divisors and remainders, to the first.

EXAMPLES.

1. Divide 1642383 by 36

6)1642383

6)273730

45621 $\frac{27}{36}$ *Quotient.*

2. Divide 15939743 by 45. *Ans.* 354216 $\frac{23}{45}$.
3. Divide 227364254 by 56. *Ans.* 4060075 $\frac{54}{56}$.
4. Divide 123456789 by 64. *Ans.* 1929012 $\frac{21}{64}$.
5. Divide 9876543210 by 72. *Ans.* 137174211 $\frac{18}{72}$.
6. Divide 4599909972 by 81. *Ans.* 56789012.
7. Divide 7680044505 by 96. *Ans.* 80000463 $\frac{57}{96}$.
8. Divide 2412121140 by 108. *Ans.* 22334455.
9. Divide 17777777616 by 144. *Ans.* 123456789.
10. Divide 414814811189 by 336.

Ans. 1234567890 $\frac{149}{336}$.

EXAMPLE.

Divide 203525 by 360

6)203525

*6)33920—5

10)5653—2

565—3

Ans. 565 $\frac{253}{360}$.

*6 *the last divisor but one.*
Multiply by 3 *the last remainder.*

18
add 2 *the second remainder.*

20
multiply by 6 *the first divisor.*

120
add 5 *the first remainder.*

125 *the true remainder.*

PROBLEMS,

Which result from a Comparison of the preceding Rules.

PROBLEM 1.

Having the least of two numbers, and their difference given, to find the greater.

RULE. Add them together.

EXAMPLE.

Let 320 be the least of two numbers, and their difference 265; what is the greater?

320
265

585 *Answer.* Or, 320+265=585.

PROBLEM 2.

When the greater of two numbers, and their difference are given, to find the less.

RULE. Subtract one from the other.

EXAMPLE.

Let the greater of two numbers be 6000, and their difference 1240; what is the less?

6000—1240=4760 *Ans.*

PROBLEM 3.

When the sum of two numbers, and one of them are given, to find the other.

RULE. Subtract the given number from the given sum, and the remainder will be the number desired.

EXAMPLE.

If 8000 be the sum of two numbers, and one of them be 3964; the other is requested?

8000—3964=4036 *Ans.*

PROBLEM 4.

When the divisor and quotient are given, to find the dividend.

RULE. Multiply them together.

EXAMPLE.

Let the divisor be 65 and the quotient 43; the dividend is required.

$65 \times 43 = 2795$ Ans.

PROBLEM 5.

When the dividend and quotient are given, to find the divisor.

RULE. Divide the dividend by the quotient.

COROLLARY. Hence we derive another method of proving division.

Ex. Given { 2875 the dividend. / 23 the Quotient.

Required the divisor. $2875 \div 23 = 125$ Ans.

PROBLEM 6.

Having the product of two numbers, and one of them given, to find the other.

RULE. Divide the product by the given number.

EXAMPLE.

Let the product of two numbers be 101088, and the given one 234; I demand the other?

$101088 \div 234 = 432$ Ans.

PROBLEM 7.

When the sum and difference of two numbers are given, to find those numbers.

RULE. To half the sum add half the difference, and the sum is the greater; and from half the sum take half the difference, and the remainder is the less.

EXAMPLE.

What are those two numbers, whose sum is 43210, and difference 3210?

$43210 \div 2 = 21605$ *half Sum.* $3210 \div 2 = 1605$ *half diff. Therefore,* $21605 + 1605 = 23210$ *the greater, and* $21605 - 1605 = 20000$ *the less.*

THEOREM. If any number be multiplied by 3, and divided by 2; this quotient tripled, and divided by 2:

this last quotient will be a number, equal to the first number, increased by a Ratio of two and one fourth: therefore,

Problem 8.

To find any number, of which any person thinks.

Rule. 1. Ask him to triple it; ask him whether the product be even or odd; if even, ask him to divide it by 2: but if it be odd, ask him to add 1, and then divide the sum by 2.

2. Ask him to triple the quotient; ask him, also, whether this product be even or odd; and proceed accordingly as in Rule 1.

3. Ask him, what the number of 9's is, in the result; and multiply the number by 4.

4. This product, increased by 1, when the rectangle by Rule 1 is odd, by 2, when the rectangle by Rule 2 is odd, or by 3, when the rectangles by both Rules are odd, will be the number required.

Problem 9.

When the sum of two numbers, and the difference of their squares are given, to find those numbers.

Rule. Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference: Hence, find the numbers by Problem 7.

Example.

What two numbers are those, whose sum is eight thousand, and the difference of whose squares is twelve millions, eight hundred thousand?

$12800000 \div 8000 = 1600$ *the difference;* ∴

$8000 \div 2 + 1600 \div 2 = 4800$ *the greater number, and*

$8000 \div 2 - 1600 \div 2 = 3200$ *the less.*

Problem 10.

When the difference of two numbers, and the difference of their squares are given, to find those numbers.

Rule. Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum; therefore, find the numbers by Problem 7.

EXAMPLE.

What are those two numbers, whose difference is 40, and the difference of whose squares is 4800?

$4800 \div 40 = 120$ *the sum of the numbers;* ∴

$120 \div 2 + 40 \div 2 = 80$ *the greater number, and*

$120 \div 2 - 40 \div 2 = 40$ *the less.*

APPLICATION.

1. The least of two numbers is 240000, and the difference between them is 123456; what is the greater, and sum of both?

Ans. 363456 *the greater and* 603456 *the sum of both.*

2. A man was born in the year 1760; when will he be 60 years of age? *Ans.* 1820.

3. General Washington was born in the year 1732; what is his age in 1806? *Ans.* 74.

4. Richard having 4000 marbles; gave to George 1000, to John 800, to Oliver 400, to Andrew 321, and to Edward 123; how many has Richard left?

Ans. 1356.

5. What number must be taken from 200000, that the remainder may be 123456? *Ans.* 76544.

6. What number must I divide by 17, that the quotient may be 156? *Ans.* 2652.

7. There are two numbers; the greater of which is 30 times 80, and their difference is 12 times 25; their sum and product are required.

Ans. 4500 *sum and* 5040000 *product.*

8. What number must be multiplied by 37, that the product may be 155400? *Ans.* 4200.

9. What number is that, which, if it be multiplied by the 47th part of 23500, will produce 400000?

Ans. 800.

10. There is a certain number, which being divided by 9, the quotient resulting, multiplied by 5, that product divided by 7, from the quotient, 50 being substract-

ed, and 60 added to the remainder, one fourth of the sum shall make 25: can you find the number?

Ans. 1134.

11. Two little Girls, A and B, had 1000 Chesnuts between them; but A had 160 more than B: how many had each? *Ans. A had* 580 *and B* 420.

12. Two little Boys, A and B, discoursing with each other; says A to B, can I not tell you any number of which you think? No; says B.

A. Have you thought of one?
B. Yes.
A. Triple it then.
B. I have.
A. Is the product even or odd?
B. It is odd.
A. Add one to the product and divide the sum by 2.
B. I have.
A. Triple the quotient.
B. I have.
A. Is the product even or odd?
B. It is odd.
A. Add one and divide the sum by 2.
B. I have.
A. How many 9's are there in the result?
B. Twenty one.

Now, from this arrangement, I demand the number that B thought of. *Ans.* 87.

13. A and B played at marbles, having 64 a piece at the first; but after playing several games, B, having lost some of his, would play no longer, and it was found that the difference of the squares of the numbers, which each then had, was 4096; pray, how many did B lose?

Ans. 16.

14. Said Jack to Charles, my Father gave me 35 oranges more than he gave my brother Harry, and the difference of the squares of our separate parcels was 4935: now, if you be Arithmetician enough to tell how many he gave us, each, you shall have one third of mine?

Ans. Jack's share was 88, *and Harry's* 53.

REDUCTION.

TABLES OF COIN, WEIGHT, AND MEASURE.

1. Money.

4 Farthings	make one	Penny,	*marked*	d.
12 Pence		Shilling,		s.
20 Shillings		Pound,		L.

2. Troy Weight.*

24 Grains	make	1 Pennyweight	*marked* grs. pwt.
20 Pennyweights		1 Ounce	oz.
12 Ounces		1 Pound	lb.

3. Apothecaries' Weight.†

20 Grains	make	1 Scruple,	*marked* gr. sc. or ℈.
3 Scruples		1 Dram,	dr. or ʒ.
8 Drams		1 Ounce,	oz. or ℥.
12 Ounces		1 Pound,	lb. or ℔.

4. Avoirdupois Weight.‡

16 Drams	make	1 Ounce,	*marked* dr. oz.
16 Ounces		1 Pound,	lb.
28 Pounds		1 Quarter of an hundred weight,	qr.
4 Quarter		1 Hund. weight, or 112 Pounds,	cwt.
20 Hundred weight		1 Ton,	T.

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* By this Weight are weighed Gold, Silver, Jewels, and Liquors.

† Apothecaries use this weight in compounding their medicines, but buy and sell their drugs by Avoirdupois weight. The pound and ounce of the Apothecaries' are the same as the pound and ounce Troy, only differently divided and subdivided.

‡ By this Weight are weighed all things of a coarse or drossy nature, as butter, cheese, flesh, grocery wares, and some liquids; bread, corn, &c. and all metals, except Gold and Silver.

A Barrel of Pork or Beef weighs 220lb.—A Quintal of Fish

5. Long Measure.*

3 Barley-corns....	make..	1 Inch..*marked* .. bar. in.
12 Inches		1 Foot, ... ft.
3 Feet		1 Yard, ... yd.
5½ Yards, or 16½ feet		1 Rod, Pole, or Perch, pol.
40 Rods.....		1 Furlong, ... fur.
8 Furlongs		1 Mile, ... mile.
3 Miles.....		1 League, ... leag.
60 Geographical miles, or 69½ Statute Miles,	1	Degree of a great Circle, ... deg.
360 Degrees.....		A great Circle, or Circumference of the Earth.

6. Cloth Measure.

2 Inches and a quarter..	make	1 Nail,..*marked* bar. in.
4 Nails		1 Quarter of a yard, qr.
3 Quarters of a yard		1 Flemish Ell,....F. El.
4 Quarters of a yard.....		1 Yard..... Yd.
5 Quarters of a yard.....		1 English Ell,....E. El.
6 Quarters of a yard.....		1 French Ell,....F. Ell.

7. Square Measure.†

144 Inches	make....	1 square Foot, *marked* Ft.
9 Square Feet		1 Square Yard, ... Yd.
30¼ Square yds, or 272¼ feet		1 Square Pole, ... Pl.
40 Square Poles.....		1 Rood, ... Rd.
4 Roods, or 160 square rds		1 Acre, ... Acre.
640 Acres.....		1 Square Mile, ... M.

112lb.—A Barrel of Raisins 112lb.—A Fother of Lead 19 1-2 Cwt. A Tod is 28lb.—A Wey 182lbs.—A Sack 364lb.—and a last 4368lb.

* 7 $\frac{92}{100}$ Inches make one link, 25 links make one pole, 100 links make one chain, and 10 chains make one furlong.

Long measure is used in measuring distances, or any thing else, where length only is regarded.

† By square measure, land, boards, glass, pavements, plastering, wainscoting, tiling, flooring, and every dimension of length and breadth only, are measured.

8. Solid Measure.*

1728 Inches	make	1 Foot.
27 Feet		1 Yard.
40 Feet of round Timber, or 50 feet of hewn Timber	..	1 Ton or Load.
128 Solid Feet, that is, 8 in length, 4 in breadth, and 4 in heighth.		1 Cord of Wood.
166½ Yards		1 Pole.
64000 Poles		1 Furlong.
512 Furlongs		1 Mile.

9. Wine Measure.†

2 Pints.. ..	make	1 Quart.
4 Quarts		1 Gallon.
42 Gallons		1 Tierce.
63 Gallons		1 Hogshead.
84 Gallons		1 Puncheon.
2 Hogsheads		1 Pipe or Butt.
2 Pipes		1 Tun.

10. Ale and Beer Measure.‡

2 Pints	make	1 Quart, *marked*	pts. qts.
4 Quarts		1 Gallon	gal.
8 Gallons		1 Firkin of Ale ..	A. fir.
9 Gallons		1 Firkin of Beer	B. fir.
2 Firkins		1 Kilderkin	Kil.
2 Kilderkins		1 Barrel	Bar.
3 Kilderkins, or 54 Gallons		1 Hogshead	Hhd.
2 Hogsheads, or 3 Barrels		1 Butt	Butt.

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* All things, that have length, breadth, and depth, are measured by this scale.

† The wine gallon contains 231 solid inches.

By this measure, brandy, spirits, cider, vinegar, oil, &c. are measured.

‡ The ale gallon contains 282 solid inches.

Milk is measured by this measure.

11. Dry Measure.*

2 Pints ..	make ..	1 Quart *marked*	pts. qts.
4 Quarts		1 Gallon	gal.
2 Gallons		1 Peck	peck.
4 Pecks		1 Bushel	bu.
8 Bushels		1 Quarter	qr.
4½ Quarter, or 36 bus.		1 Chaldron in London	ch.
5 Quarters		1 Wey	wey.
2 Weys		1 Last	Last.

12. Time.†

60 Seconds..	make..	1 Minute, *marked*	s. m.
60 Minutes		1 Hour	h.
24 Hours		1 Day	d.
7 Days		1 Week	w.
4 Weeks . . .		1 Month . . .	m.
13 Months, 1 day, and 6 hours, or 365 days, and 6 hours		1 Julian Year	Yr.

13. Motion.

60 Seconds	make	1 Prime minute, *marked*	′
60 Minutes		1 Degree	°
30 Degrees		1 Sign	s.
12 Signs, or 360 degrees		The whole great Circle of the Zodiac.‡	

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* By this Measure, corn, seeds, salt, fruit, &c. and all dry goods are measured.

† By the Calendar, the year is divided as follows.

Thirty days has September, April, June, and November;
February has twenty eight alone, and all the rest have thirty one;
But Leap-Year, coming once in four, does give to February one day more.

When ever you divide the year of our Lord by 4, without a remainder, it is then Bissextile, or Leap-Year, in which February has 29 days.

‡ The Zobiac, which is a great Circle of the Sphere, sixteen degrees broad, comprehending the orbits of all the Planets, is divided into twelve equal parts, called Signs, through which the Sun passes.

Reduction is the method of bringing numbers of one name or denomination to another, retaining the same value.

It is of two kinds, *viz.* Descending and Ascending.

REDUCTION DESCENDING.

Rule.

Multiply the number in the highest denomination, by as many of the next lower as make one in that highest, and so on, till you have brought it down to the denomination which your question requires.

Examples.

1. In *L*365 6s. 9d. 2qrs. how many farthings?

	£.	s.	d.	qrs.
	*L*365	6	9	2
Multiply by	20			
	7306			
Multiply by	12			
	87681			
Multiply by	4			
Ans.	350726			

Shillings in a pound.

Shillings.

Pence in a Shilling.

Pence.

Farthings in a penny.

Farthings.

2. In *L*. 500 how many pence? *Ans.* 120000.
3. In *L*. 98 18s. 11d. ¾ how many farthings? *Ans.* 94991.
4. In *L*. 540 19s. 6d. how two-pences? *Ans.* 64917.
5. In 39 Guineas, at 28s. how many three-pences? *Ans.* 4368.
6. In 420 Pistoles, at 22s. how many groats? *Ans.* 27720.
7. In 94 Moidores, at 36s. how many sixpences? *Ans.* 6768.
8. In 360 half Johannes, at 48s. how many half-pence? *Ans.* 414720.

9. In 1000 French Crowns, at 6s8d. how many pence and farthings?

Ans. 80000 *pence, and* 320000 *farthings.*

10. In 800 Dollars, at 6s. how many shillings, groats, two-pences, pence, half-pence, and farthings?

Ans, 4800 *shillings,* 14400 *groats,* 28800 *two-pences,* 57600 *pence,* 115200 *half-pence, and* 230400 *farthings.*

REDUCTION ASCENDING.*

Rule.

Divide the given number by as many of that denomination, as make one of the next higher, and thus continue till you have brought it to that denomination which the question requires.

Examples.

1. In 350726 farthings, how many pounds?

4)350726

12)87681—2qr.

2,0)730,6—9d.

Ans. *L.* 365—6s. 9d. 2qr.

2. In 120000 pence, how many pounds *Ans. L.* 500.
3. In 94991 farthings, how many pounds? *Ans. L.* 98. 18s. 11d. ¾.
4. Bring 64917 two-pences into pounds. *Ans. L.* 540. 19s. 6d.
5. In 4368 three-pences, how many guineas, at 28s. a guinea. *Ans.* 39.
6. In 27720 groats, how many pistoles, at 22s? *Ans.* 420.
7. Bring 6768 sixpences, into moidores, at 36s. *Ans.* 94.

* Reduction Descending and Ascending prove each other.

8. Bring 414720 half-pence into half-Johannes.
Ans. 360.

9. Reduce 320000 farthings into pence and French Crowns, at 6s8d.
Ans. 80000 *pence, and* 1000 *French Crowns.*

10. Reduce 230400 farthings into half-pence, pence, two-pences, groats, shillings, and Dollars.
Ans. 115200 *half-pence,* 57600 *pence,* 28800 *two-pences,* 14400 *groats,* 4800 *shillings, and* 800 *Dollars.*

REDUCTION DESCENDING AND ASCENDING.

1. Money.

1. In *L.* 960. 18s. how many shillings and Dollars?
Ans. 19218 *shillings and* 3203 *Dollars.*

2. In *L.* 4000. 13s. 4d. how many English Crowns, at 6s8d? *Ans.* 12002.

3. Reduce 774 English or French Crowns, to Federal Dollars. *Ans.* 860.

4. In 900 Marks, each 13s. 4d. how many Nobles, each 6s. 8d.? *Ans.* 1800.

5. In 84 Pistoles, how many pounds? *Ans. L.* 92. 8s.

6. In *L.* 600 how many Guineas?
Ans. 428 *Guineas, and* 16s. *over.*

7. In *L.* 864 how many Moidores. *Ans.* 480 *Moidores.*

8. In 4800 Guineas, how many half-Johannes?
Ans. 2800.

9. In *L.* 451. 14s. 8d. how many half-Joes, moidores, guineas, pistoles, marks, English crowns, dollars, shillings, and groats, of each, the same number?
Ans. 56 *of each sort.*

2. Troy Weight.

1. Reduce 34lb. 10oz. 16pwt. 18grs. to grains.
Ans. 201042grs.

2. In 804168grs. how many lbs.
Ans. 139lbs. 7oz. 7pwts.

3. How many Rings, each weighing, 5pwt. 9gr. may be made of 5lb. 4oz. 15qwts 9gr. of gold.
Ans. 241.

4. In 25lb. of silver, how many spoons, each weighing 6oz. and 8pwt.
Ans. 46 *spoons, and* 5oz. 12pwt. *over.*

3. Apothecaries Weight.

1. In 12℔. 9℥. 7℥. 1℈. 18grs. how many grains?
Ans. 73898.

2. In 36949 grains how many pounds?
Ans. 6℔. 4℥. 7℥. 2℈. 9gr.

4. Avoirdupois Weight.

1. Reduce 15cwt. 2gr. 14lb. 10oz. 15dr. to drams.
Ans. 448175.

2. In 448175 drams, how many hundreds?
Ans. 15cwt. 2gr. 14lb. 10oz. 15dr.

3. In 20T. 18cwt. 3qr. 27lb. 10oz. 14dr. how many dr?
Ans. 12013486 *drams.*

4. In 365 bags of Coffee, each weighing 127lb. how many Tons? *Ans.* 20T. 13cwt. 3qr. 15lb.

5. Long Measure.

1. In 1 league how many inches? *Ans.* 190080.
2. How many barley corns will reach round the world, which is 360 degrees? *Ans.* 4755801600.
3. How many times will a wheel, which is 16 feet and 6 inches in circumference, turn round in going 200 miles? *Ans.* 64000.

6. Cloth Measure.

1. In 27yd. 3qr. 2na. how many nails? *Ans.* 446 *nails.*
2. Reduce 300yd. to Flemish ells. *Ans.* 400 *F. ells.*
3. In 3560 nails, how many English ells?
Ans. 178 *E. ells.*

4. In 40 pieces of cloth, each 20 English ells. 3qr. how many American yards? *Ans.* 257½ *yd.*

7. Square Measure.

1. In 40 Acres, how many perches?
Ans. 6400 *perches.*
2. In 24114 poles, how many Acres?
Ans. 150Acres. 2r. 34po.
3. How many square feet, in a square Mile.
Ans. 27878400 *sq. feet.*

8. Solid Measure.

1. In 20 tons of round timber, how many solid inches?
Ans. 1382400.
2. In 30 tons of hewn timber, how many solid inches?
Ans. 2592000.
3. In 24 cords of wood, how many solid inches?
Ans. 5308416.

9. Wine Measure.

1. In 10hhds. 60gal. 2qts. how many quarts?
Ans. 2762.
2. In 40 tuns of wine, how many pints?
Ans. 80640 *pints.*
3. In 161280 pints, how many pipes? *Ans.* 160.
4. In 48 puncheons of rum, how many gallons?
Ans. 4032.

10. Ale and Beer Measure.

1. In 64hhds. of beer, how many pints?
Ans. 27648 *pints.*
2. In 180hhds. of beer, how many barrels? *Ans.* 270.

11. Dry Measure.

1. In 80 bushels, how many pints? *Ans.* 5120 *pts.*
2. In 256000 quarts of wheat, how many quarters?
Ans. 1000.

3. In 54 chaldrons of coals, how many bushels ?
Ans. 1944.

12. Time.

1. How many seconds are there in a year ?
Ans. 31557600.
2. In 504000 minutes how many weeks? *Ans.* 50.
3. How many seconds are there from the commencement of the christian Aera to the year 1806 ?
Ans. 56993025600.

13. Motion.

1. In 10 signs, 40°. 27'. 35". how many seconds?
Ans. 1225655.
2. In 245131" how many signs?
Ans. 2s. 8°. 5'. 31".

COMPOUND ADDITION.

Compound Addition is the adding of numbers together, which are of different denominations.

Rule.*

Place the numbers in such order, that those of the same denomination may stand directly under one another: add the first column or lowest denomination together as in simple addition; then divide the sum by as many of the same denomination as make one of the next higher; set down the remainder under the column, added, and carry the quotient to the next superior denomination, and thus continue to the last.

* The method of proof is the same as in simple addition.

1. Money.

Examples.

1.

L.	*s.*	*d.*	*qr.*
16	13	4	1
36	10	6	2
41	16	7	3
54	18	9	—
60	4	4	3
34	12	5	1

2.

L.	*s.*	*d.*	*qr.*
123	19	11	1
600	14	6	3
321	—	10	2
456	17	9	—
789	13	—	1
144	—	—	—

3.

L.	*s.*	*d.*
31	13	$6\frac{1}{4}$
14	17	$7\frac{1}{2}$
18	18	$10\frac{1}{4}$
28	19	$11\frac{3}{4}$
47	13	$8\frac{1}{2}$
59	12	7

4.

L.	*s.*	*d.*
123	11	11
456	9	$10\frac{1}{4}$
78	8	$9\frac{3}{4}$
9	16	7
123	14	$5\frac{1}{2}$
45	18	$8\frac{3}{4}$

5.

L.	*s.*	*d.*
456	15	$9\frac{3}{4}$
7	16	10
—	9	$11\frac{3}{4}$
136	14	$4\frac{1}{2}$
365	6	6
10	19	$11\frac{3}{4}$

6.

L.	*s.*	*d.*
4327	1	10
450	16	$7\frac{1}{4}$
1234	—	$\frac{3}{4}$
76	18	3
3652	3	$5\frac{1}{2}$
84	14	$\frac{1}{4}$

2. Troy Weight.

1.

lb.	*oz.*	*pwts.*	*gr.*
647	11	13	20
356	8	17	14
117	10	—	21
338	9	18	23
425	11	19	10
800	—	4	—

2.

lb.	*oz.*	*pwts.*	*gr.*
113	3	13	14
234	8	14	20
140	11	19	23
43	—	16	14
6	11	12	12
145	10	15	19

3. Apothecaries' Weight.

1.

lb.	ʒ.	ʒ.	℈.	gr.
1	11	7	2	19
2	4	6	–	14
3	10	5	1	13
4	7	4	2	16
5	4	3	–	7
6	–	6	1	12

2.

lb.	ʒ.	ʒ.	℈.	gr.
12	6	4	1	16
10	7	3	2	17
9	11	1	1	13
8	10	–	–	18
6	6	6	1	19
4	1	3	–	3

4. Avoirdupois Weight.

1.

Cwt.	*qrs.*	*lb.*	*oz.*
10	1	14	10
9	3	17	11
14	1	20	15
16	0	18	13
12	3	12	12
18	2	11	8

2.

T.	*Cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
36	17	1	27	13	14
43	16	3	14	10	11
61	6	2	16	6	12
27	18	1	13	11	9
56	12	0	18	9	10
48	8	2	23	8	7

5. Long Measure.

1.

Lg.	*mi.*	*fur.*	*pol.*	*yd.*
1	2	7	30	4
2	1	6	27	5
6	–	5	16	3
7	2	4	24	2
8	1	3	36	1
9	0	4	27	3

2.

Deg.	*mil.*	*fur.*	*pol.*	*yd.*	*ft.*	*in.*	*br.*
34	37	4	36	4	2	11	2
48	59	3	25	3	1	6	1
60	48	1	20	2	0	7	0
23	54	0	19	0	2	8	2
58	20	7	31	1	1	9	0
78	13	4	14	3	0	10	1

6. Cloth Measure.

1.				2.		3.			4.		
Yd.	*gr.*	*na.*	*in*	*Fl.*	*E.qr.*	*E.E.*	*qr.*	*na.*	*Fr.E.*	*qr.*	*na.*
47	3	1	2	127	2	16	4	3	247	5	3
36	2	1	1	34	1	17	3	1	48	3	1
23	1	3	0	6	0	12	2	2	360	4	2
30	2	1	1	14	2	21	4	0	89	2	0
16	0	2	1	100	0	30	3	3	141	4	2
18	3	0	2	7	1	14	0	2	96	2	3

7. Square Measure.

1.			2.					
Pol.	*ft.*	*in.*	*Acres.*	*r.*	*pol.*	*yd.*	*ft.*	*in.*
30	200	127	114	3	30	3	8	100
24	144	134	100	2	37	14	7	112
18	216	38	214	1	29	20	4	130
26	49	56	136	2	38	30	5	58
13	87	64	334	1	12	22	2	37
17	187	110	432	0	36	18	3	4

8. Solid Measure.

1.			2.		
Ton.	*ft.*	*in.*	*Cords.*	*ft.*	*in.*
143	16	1428	100	120	48
360	23	360	96	114	1274
212	14	1225	160	37	84
128	19	1382	83	109	1416
193	21	63	400	46	300
347	8	8	91	38	64

3. Apothecaries' Weight.

1.

lb.	ʒ.	ʒ.	℈.	gr.
1	11	7	2	19
2	4	6	–	14
3	10	5	1	13
4	7	4	2	16
5	4	3	–	7
6	–	6	1	12

2.

lb.	ʒ.	ʒ.	℈.	gr.
12	6	4	1	16
10	7	3	2	17
9	11	1	1	13
8	10	–	–	18
6	6	6	1	19
4	1	3	–	3

4. Avoirdupois Weight.

1.

Cwt.	*qrs.*	*lb.*	*oz.*
10	1	14	10
9	3	17	11
14	1	20	15
16	0	18	13
12	3	12	12
18	2	11	8

2.

T.	*Cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
36	17	1	27	13	14
43	16	3	14	10	11
61	6	2	16	6	12
27	18	1	13	11	9
56	12	0	18	9	10
48	8	2	23	8	7

5. Long Measure.

1.

Lg.	*mi.*	*fur.*	*pol.*	*yd.*
1	2	7	30	4
2	1	6	27	5
6	–	5	16	3
7	2	4	24	2
8	1	3	36	1
9	0	4	27	3

2.

Deg.	*mil.*	*fur.*	*pol.*	*yd.*	*ft.*	*in.*	*br.*
34	37	4	36	4	2	11	2
48	59	3	25	3	1	6	1
60	48	1	20	2	0	7	0
23	54	0	19	0	2	8	2
58	20	7	31	1	1	9	0
78	13	4	14	3	0	10	1

6. Cloth Measure.

1.				2.		3.			4.		
Yd.	*gr.*	*na.*	*in*	*Fl.*	*E.qr.*	*E.E.*	*qr.*	*na.*	*Fr.*	*E.qr.*	*na.*
47	3	1	2	127	2	16	4	3	247	5	3
36	2	1	1	34	1	17	3	1	48	3	1
23	1	3	0	6	0	12	2	2	360	4	2
30	2	1	1	14	2	21	4	0	89	2	0
16	0	2	1	100	0	30	3	3	141	4	2
18	3	0	2	7	1	14	0	2	96	2	3

7. Square Measure.

1.			2.					
Pol.	*ft.*	*in.*	*Acres.*	*r.*	*pol.*	*yd.*	*ft.*	*in.*
30	200	127	114	3	30	3	8	100
24	144	134	100	2	37	14	7	112
18	216	38	214	1	29	20	4	130
26	49	56	136	2	38	30	5	58
13	87	64	334	1	12	22	2	37
17	187	110	432	0	36	18	3	4

8. Solid Measure.

1.			2.		
Ton.	*ft.*	*in.*	*Cords.*	*ft.*	*in.*
143	16	1428	100	120	48
360	23	360	96	114	1274
212	14	1225	160	37	84
128	19	1382	83	109	1416
193	21	63	400	46	300
347	8	8	91	38	64

which, subtract the lower number, remembering, always, to carry one to the next superior denomination in the less number, for that which you borrowed; and thus go on to the last.*

EXAMPLES.

1. MONEY.

	1			2.		
	L.	*s.*	*d.*	*L.*	*s.*	*d.*
From	416	16	8	560	10	$8\frac{1}{2}$
Take	137	14	9	365	16	$8\frac{1}{4}$
Rem.						
Proof.						

	3.			4.			
	L.	*s.*	*d.*	*L.*			
Borrowed	1800	8	4	Lent	1100		
Paid	804	16	$4\frac{1}{2}$	Recv'd	96	13	$8\frac{1}{4}$
Remains				*Due.*			

	5.				6.		
	L.	*s.*	*d.*		*L.*		
Borrowed	6543	18	9	Lent	40000		
Paid at different times	300	14	3	Recv'd at sund times.	400	13	4
	196	10	6		2009	6	8
	2400	13	8		890	18	$6\frac{1}{2}$
	850	15	6		4000	14	$4\frac{1}{2}$
	240	19	7		444	10	$10\frac{1}{4}$
Paid in all				*Rec. in all*			
Remains to be paid				*Rem. due.*			

* The method of proof is the same as in simple subtraction.

2. Troy Weight.

1				2.				3.			
lb.	*oz.*	*pwt.*	*gr.*	*lb.*	*oz.*	*pwt.*		*lb.*	*oz.*	*pwt.*	*gr.*
17	3	14	10	100	10	13		114	9	11	12
6	0	7	20	94	11	14		67	10	16	13

3. Apothecaries' Weight.

1.					2.					3.			
lb.	℥.	ʒ.	℈.	*gr.*	lb.	℥.	ʒ.	℈.	*gr.*	lb.	℥.	ʒ.	℈.
14	7	4	1	13	23	11	4	0	11	114	8	6	1
7	6	3	0	16	14	10	7	1	13	64	7	7	2

4. Avoirdupois Weight.

1.				2.					
Cwt.	*qr.*	*lb.*	*oz.*	*Tons.*					
18	1	17	10	1000					
8	2	13	14	144	13	2	24	10	12

5. Long Measure.

1.					2.							
Lg.	*m.*	*fur.*	*pol.*	*ft.*	*Deg.*	*mi.*	*fur.*	*pol.*	*yd.*	*ft.*	*in.*	*br.*
30	1	4	36	12	300	40	3	30	1	2	5	1
14	2	3	37	16	180	60	7	15	4	1	6	2

6. Cloth Measure.

1.			2.			3.				4.		
Fl.E.	*qr.*	*na.*	*yd.*	*qr.*	*na.*	*En.E.*	*qr.*	*na.*	*in.*	*Fr.E.*	*qr.*	*na.*
48	0	3	112	1	2	300	4	3	1	140	3	1
27	2	3	39	3	3	200	4	3	2	83	4	3

7. Square Measure.

1.			2.			3.				
A.	*R.*	*pol.*	*A.*			*A.*	*R.*	*pol.*	*ft.*	*in.*
40	3	30	100			300	1	13	36	100
13	0	36	39	3	14	37	2	20	125	140

8. Solid Measure.

1.			2.			3.		
Tons.	*ft.*	*in.*	*Yd.*	*ft.*	*in.*	*Cords.*	*ft.*	*in.*
120	30	1000	60	12	112	300	100	101
24	36	1146	35	13	360	125	126	1700

9. Wine Measure.

1.			2.			3.			
Tier.	*gal.*	*qts.*	*Hhd.*	*gal.*	*qts.*	*Tuns.*	*hhd.*	*gal.*	*qts.*
86	13	1	400	18	1	800	0	21	2
47	10	3	89	36	2	700	1	36	3

10. Ale and Beer Measure.

1.				2.					3.			
B.Bar.	*fir.*	*gal.*		*B.Bar.*	*fir.*	*gal.*	*qts.*		*Hhd.*	*gal.*	*qts.*	*pts.*
60	2	5		144	1	7	1		360	41	2	1
30	2	7		78	2	8	2		187	53	2	1

11. Dry Measure.

1.			2.				3.			
Bu.	*pk.*	*qts.*	*Qu.*	*bu.*	*p.*	*qts.*	*Chal.*	*bu.*	*p.*	*qts.*
213	1	0	400	3	1	1	365	17	2	1
163	2	1	165	4	2	1	217	18	2	1

12. TIME.

1.				2.						
Mo.	*da.*	*h.*	*m.*	*Y.*	*m.*	*w.*	*d.*	*h.*	*m.*	*s.*
12	20	12	30	100	8	2	4	16	13	14
6	22	13	15	60	10	2	6	20	29	30

13. MOTION.

1.			2.			3.			
°	′	″	°	′	″	*s.*	°	′	″
180	30	40	90	11	12	8	15	12	14
90	45	15	60	40	30	4	20	50	47

COMPOUND MULTIPLICATION.

Compound Multiplication is the finding of the amount of any number of diverse denominations, repeated a certain number of times.

CASE 1.

When the multiplier does not exceed 12.

RULE.*

Place the multiplier under the lowest denomination of the multiplicand: multiply as in simple multiplication, and carry as in compound addition.†

* The general rule is, to multiply the price by the quantity.
† The method of proof is the same as in simple multiplication.

EXAMPLES.

1			2.			3.		
L.	*s.*	*d.*	*L.*	*s.*	*d.*	*L.*	*s.*	*d.*
16	14	4	31	10	10	15	13	$4\frac{1}{2}$
		2			3			4
33	8	6						

4.			5.			6.		
L.	*s.*	*d.*	*L.*	*s.*	*d.*	*L.*	*s.*	*d.*
9	11	9	18	18	$6\frac{1}{4}$	21	15	$11\frac{1}{4}$
		5			6			7

7. What will 8lb. of coffee come to, at 2*s.* 3*d.* a pound?
Ans. 18*s.*

8. 9lb. of chocolate, at 1*s.* $10\frac{1}{2}$*d.* a lb.
Ans. 16*s.* $10\frac{1}{2}$*d.*

9. 10 yd linen, at 4*s* $6\frac{1}{4}$*d.*
Ans. *L.*2 5*s.* $7\frac{1}{2}$*d.*

10. 11 yd of satin, at 9*s.* $9\frac{1}{4}$*d.*
Ans. *L.*5 7*s.* $5\frac{1}{4}$*d.*

11. 12 gallons of brandy, at 9*s.* 10*d.*
Ans. *L.*5 18*s.*

CASE 2.

When the multiplier exceeds 12, *and is a composite number.*

RULE.

Multiply the multiplicand by one of those numbers, as in Case 1; that product by another, and so on; and the last product will be the answer.

EXAMPLES.

1. What will 36 yards of cloth cost, at 4*s*. 3½*d* a yard?

	L.	*s.*	*d.*
	0	4	3½
Multiply by			6×6=36.
Produces the	1	5	9 *price of* 6 *yards.*
Multiply by			6
Ans. *L.*	7	14	6 *price of* 36 *yards.*

2. What is the value of 45 yards of velvet, at 8*s*. 3*d*. a yard? Ans. *L.*18 11*s*. 3*d*.
3. 54 lb. of indigo, at 15*s*. 7¼*d*. Ans. *L.*42 2*s*. 7½*d*.
4. 64 gallons of Rum, at 7*s*. 6*d*. Ans. *L.* 24.
5. 72 gallons of molasses, at 4*s*. 8*d*. Ans. *L.* 16 16*s*.
6. 81 barrels of flour, at *L.* 3 16*s*. 5*d*. Ans. *L.* 309 9*s*. 9*d*.
7. 88 dozen dutch quills, at 1*s*. 3¾*d*. Ans. *L.* 5 15*s*. 6*d*.
8. 108 yards of India cotton, at 2*s*. 3*d*. Ans. *L.* 12 3*s*.
9. 120 quarters of rye, at *L.* 1 13*s*. 4*d*. Ans. *L.* 200.
10. 132 yards of broadcloth, at *L.* 1 2*s*. 8*d*. Ans. *L.* 149 12*s*.
11. 144 reams of paper, at 16*s*. 7½*d*. Ans. *L.* 119 14*s*.
12. 1210 yards of Irish cloth, at 2*s*. 5*d*. Ans. *L.* 146 4*s*. 2*d*.

CASE 3.

When the multiplier is not a composite number.

RULE.

Take the nearest composite number, either greater or less than the multiplier, as is most convenient, and mul-

tiply as in the last Case: then multiply the multiplicand by the difference between this number and the multiplier, & either add this product to, or subtract it from, that, found before, according as the assumed number is greater or less than the given one.

EXAMPLES.

1. What will 17 ells of holland come to, at 8*s.* $3\frac{1}{2}$*d.* an ell?

s.	*d.*
8	$3\frac{1}{2}$

$8\times2+1=17$

3	6	4
		2
6	12	8
	8	$3\frac{1}{2}$
*L.*7	0	$11\frac{1}{2}$

Or thus,

L.	*s.*	*d.*
0	8	$3\frac{1}{2}$

$6\times3-1=17$

2	9	9
		3
7	9	3
	8	$3\frac{1}{2}$

Ans. *L.*7 0 $11\frac{1}{2}$

2. 23 bushels of corn, at 5*s.* 6*d.*
Ans. *L.* 6 6*s.* 6*d.*

3. 38 bushels of wheat, at 8*s.* $5\frac{1}{2}$*d.*
Ans. *L.* 16 1*s.* 5*d.*

4. 53 yards of tabby, at 6*s.* $7\frac{1}{4}$*d.*
Ans. *L.* 17 12*s.* $2\frac{1}{4}$*d.*

5. 65 pair of silk stockings, at 10*s.* $6\frac{1}{4}$*d.*
Ans. *L.* 34 3*s.* $10\frac{1}{4}$*d.*

6. 74 quintals of fish, at *L.* 1 4*s.* $9\frac{1}{2}$*d.*
Ans. *L.* 91 14*s.* 7*d.*

7. 94 yards of cambric, at 5*s.* 9*d.*
Ans. *L.* 27 0*s.* 6*d.*

8. 117 Cwt. of raisins, at *L.* 1 2*s.* 4*d.*
Ans. *L.* 130 13*s.*

9. 151 barrels of cider, at 5*s.* 3*d.*
Ans. *L.* 39 12*s.* 9*d.*

10. 241 Cwt. of cotton, at *L.* 12 10*s.* 8*d.*
Ans. *L.* 3020 10*s.* 8*d.*

CASE 4.

When the multiplier is large.

RULE.

Reduce the highest denomination of the multiplicand to the lowest expressed in it; under which, place the multiplier, and multiply as in simple multiplication: then, reduce the product to the highest denomination, and it will be the answer required.

EXAMPLES.

1. What will 2341 Hhd. of molasses cost, at *L.* 12 13*s.* $4\frac{1}{2}d$ a Hhd?

L.	*s.*	*d.*
12	13	$4\frac{1}{2}$
20		
253		
12		
3040		
4		
12162		
2341		
12162		
48648		
36486		
24324		
4)28471242		
12)7117810$\frac{1}{2}$		
2,0)59315,0	10	

Ans. *L.* 29657 10*s.* $10\frac{1}{2}d$.

2. What will 1827 Hhd. of salt come to, at *L.* 1 16*s.* $6\frac{1}{4}d$? Ans. *L.* 3339 19*s.* $8\frac{1}{2}d$.

3. 23000 thousand feet of boards, at *L.* 2 7*s.* 6*d.* a M.
Ans. *L.* 54625.

4. 31000 yards of tow cloth, at 1*s.* $2\frac{1}{2}$*d.* a yard.
Ans. *L.* 2054 3*s.* 4*d.*

PRACTICAL QUESTIONS IN WEIGHTS AND MEASURES.

1.

	lb.	*oz.*	*pwt.*	*gr.*
Multiply	2	4	13	15
by				3

2.

	lb.	ʒ.	ʒ.	℈.	*gr.*
Multiply	3	10	3	1	16
by					4

3.

	T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
Multiply	3	14	2	27	12	10
by						5

4.

	M.	*fur.*	*pol.*	*ft.*	*in.*
Multiply	10	6	30	15	3
by					6

5.

	yd.	*qr.*	*na.*	*in.*
Multiply	147	3	2	2
by				7

6.

	Acres.	*r.*	*pol.*	*ft.*	*in.*
Multiply	300	2	31	116	132
by					8

7.

	Cords.	*ft.*	*in.*
Multiply	13	113	1231
by			9

8.

	T.	*pi.*	*hh.*	*gal.*	*qts.*	*pt.*
Multiply	4	0	1	17	3	1
by						10

	9.			10.				11.			
	B.Hhd.	*gal.*	*qts.*	*Qt.*	*bu.*	*p.*	*gal.*	*Mo.*	*d.*	*h.*	*m.*
Multiply	120	10	3	4	5	2	1	12	14	6	13
by			11				12				24

12. What is the weight of 36 hogsheads of sugar, each weighing 3Cwt. 2qr. 21 lbs?

Ans. 132*cwt.* 3*qrs.*

13. In 43 pieces of chints, each 25¼ yards, how many yards?

Ans. 1107 *yds.* 1 *qr.*

14. In a township containing 1235 lots, each lot containing 195 acres, 2 roods, and 30 poles, how many acres?

Ans. 241674 *acres*, 0 *roods*, 10 *poles.*

COMPOUND DIVISION.

Compound Division is the dividing of numbers of different denominations.

CASE 1.

When the divisor does not exceed 12.

RULE.

Divide the highest denomination of the dividend, as in Case 4th, of simple division; and if there be a remainder, reduce it to the next inferior denomination, and divide as before; and thus proceed through all the denominations of the dividend to the lowest.*

* The method of proof is the same as in simple division.

EXAMPLES.

1.

	L.	*s.*	*d.*
2)33	8	6	
L. 16	14	3	

2. Divide *L.* 94 12*s.* 6*d.* by 3. Ans. *L.* 31 10*s.* 10*d.*
3. Divide *L.* 62 13*s.* 6*d.* by 4. Ans. *L.* 15 13*s.* 4½*d.*
4. Divide *L.* 47 18*s.* 9*d.* by 5. Ans. *L.* 9 11*s.* 9*d.*
5. Divide *L.* 113 11*s.* 1½*d.* by 6. Ans. *L.* 18 18*s.* 6¼*d.*
6. Divide *L.* 152 11*s.* 10¼*d.* by 7.
Ans. *L.* 21 11*s.* 11¾*d.*
7. If 8 lb. coffee cost 18*s.* what will 1 lb. cost?
Ans. *L.* 0 2*s.* 3*d.*
8. Divide 16*s.* 10½*d.* by 9. Ans. 1*s.* 10½*d.*
9. Divide *L.* 2 5*s.* 7½*d.* by 10. Ans. 4*s.* 6¾*d.*
10. What is satin a yard, if 11 yards cost *L.* 5 7*s.* 5¾*d.*?
Ans. 9*s.* 9¼*d.*
11. If 12 gallons of brandy cost *L.* 5 18*s.* what is that a gallon?
Ans. 9*s.* 10*d.*

CASE 2.

When the divisor exceeds 12, *and is a composite number.*

RULE.

Divide successively by its component parts; and the last quotient will be the answer.

EXAMPLES.

1. What is cloth a yard, if 36 yards cost *L.* 7 14*s.* 6*d.*?

	L.	*s.*	*d.*
6)7	14	6	
6)1	5	9	
L. 0	4	3½	*the Answer.*

2. Divide *L.* 18 11*s.* 3*d.* by 45. Ans. 8*s.* 3*d.*
3. Divide *L.* 42 2*s.* 7½*d.* by 54. Ans. 15*s.* 7¼*d.*
4. Divide *L.* 24 by 64. Ans. 7*s.* 6*d.*
5. Divide *L.* 16 16*s.* by 72. Ans. 4*s.* 8*d.*
6. Divide *L.* 309 9*s.* 9*d.* by 81. Ans. *L.* 3 16*s.* 5*d.*
7. Divide *L.* 5 15*s.* 6*d.* by 88. Ans. 1*s.* 3¼*d.*
8. Divide *L.* 12 3*s.* by 108. Ans. 2*s.* 3*d.*
9. Divide *L.* 200 by 120. Ans. *L.* 1 13*s.* 4*d.*
10. Divide *L.* 149 12*s.* by 132. Ans. *L.* 1 2*s.* 8*d.*
11. Divide *L.* 119 14*s.* by 144. Ans. 16*s.* 7½*d.*
12. If 1210 yards of Irish cloth come to *L.* 146 4*s.* 2*d.* what is that a yard? Ans. 2*s.* 5*d.*

CASE 3.

When the divisor is not a composite number.

RULE.

Divide the highest denomination of the dividend, as in Case 1st. of simple division; and if there be a remainder, reduce it to the next inferior denomination, &c.

EXAMPLES.

1. If 17 ells of holland cost *L.* 7 0*s.* 11½*d.* what is that an ell?

L. *s.* *d.* *L.* *s.* *d.*

17)7 0 11½(0 8 3½ *Ans.*

20

140

136

4

12

59

51

8

4

34

34

2. Divide *L.* 6 6*s.* 6*d.* by 23. Ans. 5*s.* 6*d.*
3. Divide *L.* 16 1*s.* 5*d.* by 38. Ans. 8*s.* $5\frac{1}{2}$*d.*
4. Divide *L.* 17 12*s.* $2\frac{1}{4}$*d.* by 53. Ans. 6*s.* $7\frac{1}{4}$*d.*
5. Divide *L.* 34 3*s.* $10\frac{1}{4}$*d.* by 65. Ans. 10*s.* $6\frac{1}{4}$*d.*
6. Divide *L.* 91 14*s.* 7*d.* by 74. Ans. *L.* 1 4*s.* $9\frac{1}{2}$*d.*
7. Divide *L.* 27 1*s.* 9*d.* by 94. Ans. 5*s.* $9\frac{1}{94}$*d.*
8. Divide *L.* 130 13*s.* by 117. Ans. *L.* 1 2*s.* 4*d.*
9. Divide *L.* 39 12*s.* 9*d.* by 151. Ans. 5*s.* 3*d.*
10. Divide *L.* 3020 10*s.* 8*d.* by 241. Ans. *L.*12 10*s.* 8*d.*
11. Divide *L.* 3339 19*s.* $8\frac{1}{4}$*d.* by 1827.
Ans. *L.* 1 16*s.* $6\frac{1}{4}$*d.*
12. Divide *L.* 29657 10*s.* $10\frac{1}{2}$*d.* by 2341.
Ans. *L.* 12 13*s.* $4\frac{1}{2}$*d.*
13. Divide *L.* 54625 by 23000.
Ans. *L.* 2 7*s.* 6*d.*
14. Divide *L.* 2054 3*s.* 4*d.* by 34000.
Ans. *L.* 0 1*s.* $2\frac{1}{2}$*d*

PRACTICAL QUESTIONS IN COIN, WEIGHT, AND MEASURE.

1. Divide *L.* 107 6*s.* 8*d.* among 8 men and 7 women, and give the men twice as much as the women.

Ans. { *L.* 4 13*s.* 4*d.*=*a woman's share.*
{ *L.* 9 6*s.* 8*d.*=*a man's share.*

2. Divide *L.* 176 8*s.* 4*d.* among 4 men, 5 women, and 6 boys: give each man three times as much as a woman, and each woman twice as much as a boy.

Ans. { *L.* 4 8*s.* $2\frac{1}{2}$*d.*=*a boy's share.*
{ *L.* 8 16*s.* 5*d.*=*a woman's share.*
{ *L.* 26 9*s.* 3*d.*=*a man's share.*

3. Divide 10 guineas among 8 men: give A 9*d.* more than B, B 9*d.* more than C, &c.
Ans. H's share = *L.* 1 12*s.* $4\frac{1}{2}$*d.*

4.

lb. oz. pwt. gr.

3) 7 2 0 21

5.

℔ ℥ ʒ ℈ *gr.*

4) 15 5 6 1 4

6

T. cwt. qr. lb. oz. dr.

5) 18 13 2 26 14 2

7.

M. fur. pol. ft. in.

6) 65 0 25 8½ 6

8.

Yd. qr. n. in.

7) 1035 2 0 ½

9.

Acres. r. pol. ft. in.

8) 2405 2 11 118¼ 48

10.

Cords. ft. in.

9) 124 127 738

11.

T. p. hhd. gal. qts. pts.

10) 43 0 0 52 3 0

12.

B. Hhd. gal. qt.

11) 1322 10 1

13.

Qts. bu. p. gal.

12) 56 3 2 0

14.

M. d. h. m.

260 18 6 12 by 24.

15.

s. ° ′ ″

4 25 15 30 by 39.

VULGAR FRACTIONS.

Vulgar Fractions are expressions for any assignable parts of an unit.

1. An *unit* is that by which every thing in nature is called one.

2. *Number* is that which is composed of one or more units.

3. One number is said to *measure* another, when it divides it without leaving a remainder.

4. A *common measure* is that which measures two or more numbers; and the *greatest* number, which can do this, is called the *greatest common measure.*

5. A *multiple* is that which is measured by another number.

6. A *common multiple* is that which is measured by two or more numbers; and if it be the least number which can be so measured, it is called the *least common multiple.*

7. An *even number* is that which can be measured by 2.

8. An *odd number* is that which cannot be measured by 2, or which differs from an even number by unity.

9. A *prime number* is that which can only be measured by itself, or an unit.

10. One number is said to be *prime* to another, when unity is the only number by which they can both be measured.

11. A *composite number* is that which is produced by multiplying two or more numbers together.

12. A *perfect number* is that which is equal to the sum of all its *aliquot* parts.

Vulgar Fractions are represented by two numbers, placed one above the other, with a line drawn between them.

The figures, above the line, are called the *numerator*, and those below it, the *denominator.*

The denominator (which is the divisor in division) shews how many parts into which the integer is divided; and the numerator (which is the remainder after division) shews how many of those parts are designed by the fraction.

Vulgar Fractions are either *proper*, *improper*, *single*, or *compound*.

1. A *proper fraction* is when the numerator is less than the denominator; as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{7}$, &c.

2. An *improper fraction* is when the numerator exceeds the denominator; as, $\frac{4}{3}$, $\frac{7}{4}$, &c.

3. A *single fraction* is a simple expression denoting any number of parts of the integer.

4. A *compound fraction* is the fraction of a fraction; as, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{7}$, &c.

5. A *mixed number* is that which is composed of a whole number and a fraction; as, $4\frac{1}{2}$, $10\frac{4}{7}$, &c.

PROBLEM 1.

To find the greatest common measure of two or more numbers.

Rule.

If there be two numbers only, divide the greater by the less; this divisor by the remainder, and so on, till nothing remains, then will the last divisor be the greatest common measure required. But, when there are more than two numbers, find the greatest common measure of two of them as before; and of that common measure and one of the other numbers, and so on, through all the numbers to the last; then will the greatest common measure, last found, be the answer.*

---

* If 1 is found to be the common measure, the given numbers are *prime* to each other, and consequently are in their lowest terms.

EXAMPLES.

1. What is the greatest common measure of 518, 2294, and 703?

518)2294(4
2072

222)518(2
444

74)222(3
222

...

Common measure.

So 74 is the greatest common measure of 518 and 2294.

Hence 74)703(9
666

37)74(2
74

..

Last greatest common measure

Therefore 37 is the answer required.

2. What is the greatest common measure of 2688 and 9936? *Ans.* 48.

3. What is the greatest common measure of 2304, 2784, and 936? *Ans.* 24.

4. The greatest common measure of 9216, 11136, 936, and 136 is required. *Ans.* 8.

PROBLEM 2.

To find the least common multiple of two or more numbers.

RULE.

Divide by any number that will measure two or more of the given numbers, and set the quotients, together with the undivided numbers, in a right line below them:

Divide the second line as before, and so on, till there are no two numbers that can be measured; then, the continued product of the divisors and quotients will give the multiple required.

EXAMPLES.

1. What is the least common multiple of 5, 20, and 24?

4)5, 20, 24

5)5, 5, 6

1, 1, 6

$4 \times 5 \times 6 = 120$ *the Answer.*

2. What is the least common multiple of 8 and 10?
Ans. 40.
3. The least common multiple of 3, 4, and 6 is required?
Ans. 12.
4. What is the least number that 3, 4, 8, and 9 will measure? *Ans.* 72.
5. What is the least common multiple of 2, 3, 4, 5, 6, and 7? *Ans.* 420.
6. What is the least number that can be divided by the 9 digits, separately, without a remainder?
Ans. 2520.

REDUCTION OF VULGAR FRACTIONS.

Reduction of Vulgar Fractions is the bringing of them out of one form or denomination into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE 1.

To abbreviate, or reduce fractions to their lowest terms.

RULE 1.

Divide the terms of the given fraction by any number that will measure them, and these quotients again in the

same manner; and so on, till there is no number *greater* than 1, that can measure them, and the fraction will be in its lowest terms.

RULE 2.

Find the greatest common measure by Problem 1, and by it, divide both the terms of the given fraction, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{144}{180}$ to its lowest terms.

(4) (3) (3)

By Rule 1. $\frac{144}{180}=\frac{36}{45}=\frac{12}{15}=\frac{4}{5}$, the Answer.

By Rule 2. 144)180(1
144
---
36)144(4
144

...

Therefore 36 *is the greatest common measure, and* $36)\frac{144}{180}=\frac{4}{5}$, *the same as before.*

2. Reduce $\frac{2346}{4691}$ to its lowest terms. *Ans.* $\frac{1}{2}$.
3. Reduce $\frac{245}{735}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
4. Reduce $\frac{1370}{19108}$ to its lowest terms. *Ans* $\frac{5}{4}$.
5. Reduce $\frac{8638}{9872}$ to its lowest terms. *And.* $\frac{7}{8}$.
6. Reduce $\frac{615}{2117}$ to its lowest terms. *Ans.* $\frac{5}{17}$.
7. Reduce $\frac{202223}{205444}$ to its lowest terms. *Ans.* $\frac{631}{641}$.
8. Reduce $\frac{1824}{91200}$ to its lowest terms. *Ans.* $\frac{1}{50}$.
9. Reduce $\frac{1386240}{4106180}$ to its lowest terms. *Ans.* $\frac{304}{9005}$.

CASE 2.

*To reduce a whole number to an equivalent fraction, having a given denominator.**

RULE.

Multiply the whole number by the given denominator:

---

* Fractions may be reduced to others, whose numerators or denominators are given, by these two Cases following.

place the product over the said denominator, and it will form the fraction required.

Corollary. Hence, any whole number may be reduced to a fraction by writing unity under it.

EXAMPLES.

1. Reduce 5 to a fraction, whose denominator shall be 8. $5 \times 8 = 40$; $\therefore$ $\frac{40}{8}$ *the Ans.*
2. Reduce 8 to a fraction, whose denominator shall be 9. *Ans.* $\frac{72}{9}$.
3. Reduce 80 to a fraction, whose denominator shall be 40. *Ans.* $\frac{3200}{40}$.
4. Reduce 200 to a fraction, whose denominator shall be 300. *Ans.* $\frac{60000}{300}$.

CASE 3.

To reduce a mixed number to its equivalent improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction; to the product add its numerator; and the de-

---

CASE 1.

To reduce fractions to others of the same value, having the numerator of the required fraction given.

RULE.

As the numerator of the given fraction, is to its denominator; so is the numerator of the required fraction, to its denominator.

nominator written under the sum, will form the fraction required.

EXAMPLES.

1. Reduce $23\frac{3}{4}$ to its equivalent improper fraction.

$23\frac{3}{4}$

$4+3$

95 ∴ $\frac{95}{4}$ *is the Ans. Or,* $\frac{23\times4+3}{4} = \frac{95}{4}$ *Ans. as before.*

2. Reduce $46\frac{7}{8}$ to its equivalent improper fraction.

Ans. $\frac{375}{8}$.

3. Reduce $48\frac{4}{19}$ to its equivalent improper fraction.

Ans. $\frac{916}{19}$.

EXAMPLES.

1. Reduce $\frac{3}{5}$ to a fraction of the same value, whose numerator shall be 12.

n. d. n. d.

As 3 : 5 : : 12 : 20; ∴ $\frac{12}{20}$ *is the answer, which is equal to* $\frac{3}{5}$, *the given fraction.*

2. Reduce $\frac{3}{4}$ to an equivalent fraction, whose numerator shall be 34.

Ans. $\frac{34}{45\frac{1}{3}}$

CASE 2.

To reduce fractions to others of the same value, having the denominator of the required fraction given.

4. Reduce $100\frac{11}{14}$ to an improper fraction.

Ans. $\frac{1411}{14}$.

5. Reduce $240\frac{121}{144}$ to an improper fraction.

Ans. $\frac{34681}{144}$.

6. Reduce $251\frac{1234}{5000}$ to an improper fraction.

Ans. $\frac{1406834}{5000}$.

CASE 4.

To reduce an improper fraction to its equivalent whole or mixed number.

RULE.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

RULE.

As the denominator of the given fraction, is to its numerator; so is the denominator of the required fraction, to its numerator.

EXAMPLES.

1. Reduce $\frac{12}{20}$ to a fraction of the same value, whose denominator shall be 5.

As 20 : 12 : : 5 : 3; ∴ $\frac{3}{5}$ *is the Answer required.*

2. Reduce $\frac{7}{8}$ to another fraction of the same value, whose denominator shall be 35.

Ans. $\frac{30\frac{5}{8}}{35}$

Corollary. From these two Cases arises another species of fractions, which may not improperly be called mixed fractions.

EXAMPLES.

1. Reduce $\frac{95}{4}$ to its equivalent whole, or mixed number.

4)95(23$\frac{3}{4}$ *Ans.*
8
—
15
12
—
3 *Or,* $\frac{95}{4}=95\div 4=23\frac{3}{4}$ *as before.*

2. Reduce $^{1}\frac{02}{12}$ to its equivalent whole or mixed number. *Ans.* 16.

The reduction of these mixed fractions to single is as follow.

CASE 1.

When the numerator is the integral part.

RULE.

Multiply the integral part by the denominator of the fractional part, and to the product add the numerator of the fractional, for a new numerator: then, multiply the denominator of the fraction by the denominator of the fractional part, for a new denominator.

EXAMPLE.

Reduce $\frac{30\frac{5}{8}}{35}$ to a single fraction.

$\frac{30\times 8+5}{35\times 8}=\frac{245}{280}=\frac{7}{8}$ the Answer.

3. Reduce $3\frac{7}{8}$ to its equivalent whole or mixed number. *Ans.* $46\frac{7}{8}$.
4. Reduce $9\frac{16}{19}$ to its equivalent whole or mixed number. *Ans.* $48\frac{4}{19}$.
5. Reduce $3\frac{768}{144}$ to its equivalent whole or mixed number. *Ans.* $240\frac{192}{144}$.

CASE 5.

To reduce a compound fraction to an equivalent single or simple one.

RULE.

Multiply all the numerators together for a new numerator, and all the denominators, for a new denominator, and they will form the single fraction required.

Scholium. If part of the compound fraction be a whole or mixed number, it must be reduced to a fraction either by the *Corollary* to *Case* 2*d.* or by *Case* 3*d.*

CASE 2.

When the denominator is the integral part.

RULE.

Multiply the integral part by the denominator of the fractional part, and to the product add the numerator of the fractional, for a new denominator: then, multiply the numerator of the fraction by the denominator of the fractional part, for a new numerator.

EXAMPLE.

Reduce $\frac{34}{45\frac{1}{3}}$ to a single fraction.

$$\frac{34\times3}{45\times3+1}=\frac{102}{136}=\frac{3}{4}$$ the Answer.

Examples.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to an equivalent single fraction.

$$\frac{1\times2\times3\times4\times5}{2\times3\times4\times5\times6}=\frac{120}{720}=\frac{1}{6}$$ *the Answer.*

2. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ to an equivalent simple fraction. *Ans.* $\frac{1}{4}$.
3. Reduce $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{11}{12}$ to a simple fraction. *Ans.* $\frac{11}{16}$.
4. Reduce $\frac{5}{12}$ of $\frac{15}{16}$ of $\frac{8}{17}$ of $\frac{4}{5}$ to a simple fraction. *Ans.* $\frac{5}{34}$.
5. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{6}{7}$ of 40 to a single fraction. *Ans.* $\frac{2}{7}$.
6. Reduce $\frac{6}{7}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{2}$ of $201\frac{7}{8}$ to a single fraction. *Ans.* $\frac{2016}{1680}=1\frac{336}{1680}$.

CASE 6.

To reduce fractions of different denominators to equivalent fractions having a common denominator.

Rule 1.

Multiply each numerator into all the denominators, but its own, for a new numerator, and all the denominators into one another, for a common denominator.

Examples.

1. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to equivalent fractions, having a common denominator.

$1\times3\times4=12$ *the new numerator for* $\frac{1}{2}$.
$1\times2\times4=8$ *the new numerator for* $\frac{1}{3}$.
$1\times2\times3=6$ *the new numerator for* $\frac{1}{4}$.
$2\times3\times4=24$ *the common denominator.* ∴ *The new equivalent fractions are,* $\frac{12}{24}$, $\frac{8}{24}$, *and* $\frac{6}{24}$, *the Ans.*

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to equivalent fractions, having a common denominator. *Ans.* $\frac{40}{60}$, $\frac{45}{60}$, *and* $\frac{48}{60}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to new equivalent fractions, having a common denominator.

Ans. $\frac{120}{180}$, $\frac{120}{180}$, $\frac{135}{180}$, and $\frac{150}{180}$.

4. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$, to fractions, having a common denominator.

Ans. $\frac{2520}{5040}$, $\frac{3360}{5040}$, $\frac{3780}{5040}$, $\frac{4032}{5040}$, $\frac{4200}{5040}$, and $\frac{4320}{5040}$.

5. Reduce $\frac{7}{8}$, $\frac{8}{9}$, $\frac{5}{11}$, and $\frac{4}{11}$ of 20 to a common denominator. *Ans.* $\frac{98}{112}\frac{28}{112}$, $\frac{99}{112}\frac{84}{112}$, $\frac{46}{112}\frac{80}{112}$, and $\frac{69}{112}\frac{120}{112}$.

6. Reduce $\frac{3}{7}$, $\frac{4}{5}$ of $\frac{5}{6}$, $8\frac{3}{4}$, and $\frac{1}{11}$, to a common denominator. *Ans.* $\frac{31}{40}\frac{20}{80}$, $\frac{32}{40}\frac{20}{80}$, $\frac{40}{40}\frac{50}{80}$, and $\frac{1}{40}\frac{80}{80}$.

Rule 2.

To reduce any given fractions to others, which shall have the least common denominator.

1. By Problem 2d. find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the products will be the numerators of the fractions required.

Examples.

1. Reduce $\frac{7}{8}$ and $\frac{5}{12}$ to equivalent fractions, having the least common denominator possible.

4)8, 12

2, 3 $4\times2\times3=24$=*least common denominator.*

$24\div8\times7=21$ *the first numerator, and* $24\div12\times5=10$ *the second numerator; therefore, the required fractions are* $\frac{21}{24}$ *and* $\frac{10}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to fractions, having the least common denominator. *Ans.* $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to fractions, having the least common denominator. *Ans.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$.

4. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$ to fractions, having the least common denominator.

Ans. $\frac{1260}{2520}$, $\frac{1680}{2520}$, $\frac{1890}{2520}$, $\frac{2016}{2520}$, $\frac{2100}{2520}$, $\frac{2160}{2520}$, $\frac{2205}{2520}$, and $\frac{2240}{2520}$.

CASE 7.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

RULE.

Reduce the given fraction to a compound one, by comparing it with all the denominations between *it* and *that* denomination to which you would reduce it: then, reduce the compound fraction to a single one, by Case 5th. and it will be the answer required.

EXAMPLES.

1. Reduce $\frac{4}{5}$ of a penny to the fraction of a pound.

By comparing it, according to the Rule, it becomes a compound fraction, thus, $\frac{4}{5}$ *of* $\frac{1}{12}$ *of* $\frac{1}{20}$; *which must be reduced to a single fraction, thus,*

$$\frac{4\times 1\times 1}{5\times 12\times 20}=\frac{4}{1200}=\frac{1}{300}$$ *Answer.*

2. Reduce $\frac{1}{5}$ of a farthing to the fraction of a pound.

Ans. L. $\frac{1}{4800}$.

3. Reduce $\frac{2}{3}$ of a penny to the fraction of a pistole.

Ans. $\frac{1}{360}$.

4. Reduce $\frac{5}{6}$ of a pound to the fraction of a guinea.

Ans. $\frac{25}{42}$.

5. Reduce $\frac{7}{8}$ of a pound to the fraction of a moidore.

Ans. $\frac{35}{72}$.

6. Reduce $\frac{12}{13}$ of a pistole to the fraction of a half joe.

Ans. $\frac{12}{26}$.

7. Reduce $\frac{8}{9}$ of a half joe to the fraction of an Eagle.

Ans. $\frac{32}{45}$.

8. Reduce $\frac{3}{4}$ of a grain to the fraction of a pound Troy.

Ans. $\frac{1}{7680}$.

9. Reduce $\frac{5}{8}$ of a Cwt. to the fraction of a Ton.
Ans, $\frac{1}{32}$.

10. Reduce $\frac{20}{21}$ of a rod to the fraction of a league.
Ans. $\frac{1}{1008}$.

11. Reduce $\frac{4}{7}$ of a nail to the fraction of a yard.
Ans. $\frac{1}{28}$.

12. Reduce $\frac{5}{8}$ of a pole to the fraction of an acre.
Ans. $\frac{1}{256}$.

13. Reduce $\frac{4}{9}$ of a foot to the fraction of a cord.
Ans. $\frac{1}{288}$.

14. Reduce $\frac{9}{13}$ of a pint of wine to the fraction of a tun.
Ans. $\frac{1}{1872}$.

15. Reduce $\frac{14}{15}$ of a gallon of beer to the fraction of a hhd.
Ans.. $\frac{7}{405}$.

16. Express $\frac{3}{4}$ of a peck in the fraction of a quarter.
Ans. $\frac{1}{32}$.

17. Reduce $\frac{3}{5}$ of a day to the fraction of a month.
Ans. $\frac{1}{10}$.

18. Reduce $\frac{15}{16}$ of a prime minute to the fraction of a sign.
Ans. $\frac{1}{1296}$.

CASE 8.*

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

Rule.

Reduce the given fraction to a compound one by comparing it, in an inverted order, with all the denominations as before; likewise, reduce the compound fraction to a single one, and it will be the fraction required.

---

* This case and case 7th. prove each other.

Examples.

1. Reduce $\frac{1}{100}$ of a pound to the fraction of a penny.

According to the Rule, the comparison must stand thus, $\frac{1}{100}$ *of* $\frac{20}{1}$ *of* $\frac{12}{1}$, *then* $\frac{1\times20\times12}{300\times1\times1}=\frac{240}{100}=\frac{4}{5}$ *Ans.*

2. Reduce *L.* $\frac{1}{1000}$ to the fraction of a farthing. Ans $\frac{4}{5}$.
3. Reduce $\frac{1}{100}$ of a pistole to the fraction of a penny. Ans. $\frac{2}{1}$.
4. Reduce $\frac{21}{48}$ of a guinea to the fraction of a pound. Ans. *L.* $\frac{7}{8}$.
5. Reduce $\frac{11}{14}$ of a moidore to the fraction of a pound. Ans. *L.* $\frac{7}{8}$.
6. Reduce $\frac{1}{20}$ of a half joe to the fraction of a pistole. Ans. $\frac{13}{17}$.
7. Reduce $\frac{13}{45}$ of an eagle to the fraction of a half joe. Ans. $\frac{8}{9}$.
8. Reduce $\frac{1}{7000}$ of a pound Troy to the fraction of a grain. Ans. $\frac{1}{4}$.
9. Reduce $\frac{1}{32}$ of a ton to the fraction of a Cwt. Ans. $\frac{5}{8}$.
10. Reduce $\frac{1}{1000}$ of a league to the fraction of a pole. Ans. $\frac{20}{51}$.

CASE 9.

To find the value of a fraction in the known parts of the integer.

Rule.

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator: if there be a remainder, multiply it by the next inferior denomination, and divide by the denominator, as before, and so on; and the several quotients will be the answer.

EXAMPLES.

1. What is the value of $\frac{3}{7}$ of a pound?

3
20
—
7)60
—
8——4
12
—
7)48
—
6——6
4
—
7)24
—
$3\frac{3}{7}$

Ans. 8*s.* 6*d.* $3\frac{3}{7}$ *qrs.*

2. What is the value of $\frac{5}{8}$ of a pound? Ans. 12s. 6d.
3. What is the value of $\frac{4}{9}$ of L. 8. 16s? Ans. L. 3. 18s. $2\frac{2}{3}$d.
4. What is the value of $\frac{4}{5}$ of a moidore? Ans. L. 1. 8s. 9d. $2\frac{2}{5}$qr.
5. What is the value of $\frac{3}{7}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of an eagle? Ans. L. 1. 4s.
6. What is the value of $\frac{1}{7}$ of a pound Troy? Ans. 2oz. 8pwt.
7. What is the value of $\frac{4}{11}$ of a Ton? Ans. 6cwt. 0qr. 17lb. 3oz. $11\frac{1}{11}$d.
8. What is the value of $\frac{3}{20}$ of $41\frac{1}{2}$ miles? Ans. 6m. 1fur. 32r.
9. What is the value of $\frac{3}{8}$ of an English ell? Ans. 1qr. $3\frac{1}{2}$ na.
10. What is the value of $\frac{9}{10}$ of an acre? Ans. 3r. 24p.
11. What is the value of $\frac{3}{5}$ of a ton of hewn timber? Ans. 33ft, 1152in.
12. What is the value of $\frac{1}{2}$ of $\frac{1}{12}$ of a hhd. of wine? Ans. $28\frac{3}{4}\frac{1}{8}$gal.

13. What is the value of $\frac{4}{17}$ of a hhd. of ale?
Ans. 12gal. $3\frac{1}{7}$qts.

14. What is the value of $\frac{3}{4}$ of $\frac{5}{9}$ of a quarter of corn?
Ans. $3\frac{1}{3}$bush.

15. What is the value of $\frac{17}{18}$ of a Julian year?
Ans. 344d. 23ho.

16. The value of $\frac{5}{16}$ of a degree is required.
Ans. 18.' 45."

CASE 10.

To reduce any given quantity to the fraction of any greater denomination of the same kind.

Rule.

Reduce the given quantity to the lowest denomination expressed in it for a numerator; then reduce the integral part to the same term for a denominator; and they will constitute the fraction required.

Scholium. If there be a fraction in the given quantity, the quantity must be reduced to the denominative parts thereof, adding thereto the numerator.

Examples.

1. Reduce 8s. 6d, $3\frac{1}{7}$qr. to the fraction of a pound.

	s.	d.	qr.
20 Integral part.	8	6	$3\frac{1}{7}$
12	12		
240	102		
4	4		
960	411		
7	7+3		
6720 Denominator.	2880 Numerator.		

Ans. $\frac{2880}{6720} = \frac{3}{7}$ L.

2. Reduce 12s. 6d. to the fraction of a pound. Ans. L. $\frac{5}{8}$.
3. Reduce L. 3. 18s. $2\frac{2}{7}$d. to the fraction of L. 8. 16s. Ans. $\frac{4}{9}$.
4. Reduce L. 1. 8s. 9d. $2\frac{1}{7}$qr. to the fraction of a moidore. Ans. $\frac{4}{7}$ moi.
5. Reduce L. 1. 4s. to the fraction of an Eagle. Ans. $\frac{2}{7}$ Eag.
6. Reduce 2oz. 8pwt. to the fraction of a pound Troy. Ans. $\frac{1}{7}$lb.
7. Reduce 6cwt. 0qr. 17lb. 3oz. $11\frac{1}{7}$dr. to the fraction of a ton. Ans. $\frac{4}{17}$Ton.
8. What part of $41\frac{1}{2}$ miles is 6m. 1fur. 32r? Ans. $\frac{1}{25}$.
9. Reduce 1qr. $3\frac{1}{2}$na. to the fraction of an English ell. Ans. $\frac{3}{8}$ En. ell.
10. Reduce 3r. 24po. to the fraction of an acre. Ans. $\frac{9}{160}$ Acre.
11. Reduce 33ft. 1152in. of hewn timber to the fraction of a ton. Ans. $\frac{2}{3}$ T.
12. Reduce $28\frac{21}{24}$ gallons of wine to the fraction of a Hhd. Ans. $\frac{11}{24}$Hhd.
13. Reduce 12gal. $3\frac{1}{7}$qts. of ale to the fraction of a hhd. Ans. $\frac{4}{17}$Hhd.
14. Reduce $3\frac{1}{3}$ bushels of corn to the fraction of a quarter. Ans. $\frac{5}{12}$.
15. Reduce 314d. 23h. to the fraction of a Julian year. Ans. $\frac{17}{18}$.
16. Reduce 18.′ 45.″ to the fraction of a degree. Ans. $\frac{5}{16}$.

ADDITION OF VULGAR FRACTIONS.

Rule.

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLES.

1. Add $\frac{1}{2}$ and $\frac{2}{3}$ together.

$1\times3=3$, $2\times2=4$ — *or,* $\frac{1}{2}=\frac{3}{6}$ *and* $\frac{2}{3}=\frac{4}{6}$; $\therefore$ $\frac{3}{6}+\frac{4}{6}=\frac{7}{6}=1\frac{1}{6}$ *sum as before.*

$2\times3=6$, $\frac{7}{6}=1\frac{1}{6}$ *the Answer.*

2. Add $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ together. Ans. $1\frac{53}{60}$.
3. Add $\frac{1}{4}$, $\frac{2}{3}$ of 6, and $\frac{7}{8}$ together. Ans. $3\frac{21}{24}$.
4. Add $\frac{4}{9}$, 8, $7\frac{1}{4}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together. *Ans.* $16\frac{57}{108}$.
5. Add $\frac{3}{5}$, 11, $\frac{7}{9}$ of $\frac{1}{4}$, and $\frac{6}{7}$ of $\frac{7}{8}$ of $15\frac{9}{10}$ together. *Ans.* $23\frac{251}{360}$.
6. What is the sum of $\frac{5}{6}$ of $\frac{1}{3}$, $\frac{2}{5}$ of $\frac{1}{7}$ of 2, $3\frac{1}{2}$, $\frac{1}{9}$ of $\frac{1}{4}$, and $\frac{3}{10}$? *Ans.* $4\frac{347}{360}$.
7. Add *L.* $\frac{1}{2}$, $\frac{3}{4}$s. and $\frac{7}{8}$d. together. *Ans. L.* 0. 10s. 9d. $3\frac{1}{2}$qr.
8. Add $\frac{3}{4}$ of a pistole, *L.* $\frac{1}{9}$, $\frac{1}{7}$s. and $\frac{4}{7}$d. together. *Ans.* 19s. $2\frac{64}{105}$d.
9. What is the sum of $\frac{4}{9}$ of an eagle, $\frac{6}{7}$ of a moidore, $\frac{3}{5}$ of a guinea, and $\frac{6}{11}$ of a dollar ? *Ans. L.* 3. 17s. $10\frac{614}{1155}$d.
10. Add $\frac{5}{8}$ of a ton, $\frac{2}{9}$ of a Cwt. and $\frac{3}{4}$ of $\frac{4}{7}$ of a lb. together. *Ans.* 16Cwt. 3q. 16lb. 2oz. $7\frac{3}{4}\frac{7}{7}$dr.
11. What is the sum of $\frac{3}{4}$ of $\frac{5}{6}$ of a mile, $\frac{3}{4}$ of a yard, and $\frac{1}{3}$ of a foot ? *Ans.* 1100yds. 2ft. 7in.
12. Add $\frac{1}{2}$ of a week, $\frac{4}{7}$ of a day, $\frac{3}{4}$ of an hour, and $\frac{3}{7}$ of a minute together. *Ans.* 4d. 7h. 57m. 40sec.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as in addition, and the difference of the numerators, written above the common denominator, will be the difference of the fractions required.

---

* A fraction may be subtracted from a whole number more concisely, than by the general Rule, thus ; take the

Examples.

1. From $\frac{2}{3}$ take $\frac{1}{4}$

$2\times4=8$ } *Or,* $\frac{2}{3}=\frac{8}{12}$ *and* $\frac{1}{4}=\frac{3}{12}$; ∴
$1\times3=3$ } $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$ *Remainder.*

5
— *the Answer.*
$3\times4=12$

2. From $\frac{7}{8}$ take $\frac{5}{6}$. *Ans.* $\frac{11}{24}$.
3. From 9 take $\frac{6}{7}$. *Ans.* $8\frac{1}{7}$.
4. From $12\frac{1}{2}$ take $8\frac{5}{6}$. *Ans.* $3\frac{2}{3}$.
5. From $\frac{4}{5}$ of $\frac{7}{8}$ take $\frac{1}{4}$ of $\frac{5}{9}$. *Ans.* $\frac{101}{180}$.
6. From $13\frac{1}{4}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of 4. *Ans.* $12\frac{5}{12}$.
7. From $\frac{4}{7}$ of $\frac{7}{8}$ of 81 take $\frac{3}{4}$ of $\frac{1}{2}$ of $9\frac{5}{9}$. *Ans.* $36\frac{11}{12}$.
8. From *L.* $\frac{2}{9}$ take $\frac{2}{5}$s. *Ans.* 3s. 10d. $0\frac{8}{15}$qr.
9. From $\frac{8}{9}$ *L.* 2. 10s. take $\frac{4}{5}$ of $\frac{3}{4}$ of a dollar.
Ans. L. 2. 0s. $10\frac{3}{11}$d.
10. From $\frac{7}{8}$oz. take $\frac{3}{5}$pwt. *Ans.* 16pwt. $21\frac{3}{5}$gr.
11. From $\frac{6}{7}$ of a league take $\frac{5}{9}$ of a mile.
Ans. 2m. 0f. 5r. 0yd. $1\frac{11}{44}$ft.
12. From 3 weeks take $16\frac{1}{5}$ days.
Ans. 4d. 9h. 36m.

---

numerator of the fraction from its denominator, and place the remainder over the denominator, then take one from the whole number. Ex. $5-\frac{3}{8}=4\frac{5}{8}$. And likewise, mixed numbers may be subtracted from mixed numbers, when the lower fraction (the subtrahend) is greater than the upper one (the minuend) thus; subtract the numerator of the subtrahend from the common denominator, and to the difference add the numerator of the minuend, and carry one to the integer of the subtrahend. Ex. From $17\frac{2}{5}$ take $10\frac{3}{5}$—$17\frac{2}{5}-10\frac{3}{5}=6\frac{4}{5}$.

MULTIPLICATION OF VULGAR FRACTIONS.

Rule.*

Reduce compound fractions to single ones, and mixed numbers to improper fractions; then multiply all the numerators together for a numerator, and all the denominators, for a denominator, and it will be the product required.

Examples.

1. Multiply $\frac{7}{8}$ by $\frac{4}{5}$.

$$\frac{7}{8}\times\frac{4}{5}=\frac{7\times4}{8\times5}=\frac{28}{40}=\frac{14}{20}=\frac{7}{10}$$ *the Answer.*

2. Multiply $4\frac{1}{2}$ by $\frac{9}{10}$. *Ans.* $4\frac{1}{20}$.
3. Multiply $24\frac{4}{5}$ by $6\frac{1}{4}$. *Ans.* $167\frac{2}{5}$.
4. Multiply $\frac{6}{7}$ of 9 by $\frac{4}{5}$ of $\frac{2}{3}$. *Ans.* $4\frac{12}{105}$.
5. Multiply $\frac{5}{8}$ of $15\frac{5}{6}$ by $\frac{3}{7}$ of $1\frac{1}{2}$. *Ans.* $6\frac{81}{224}$.

---

* The multiplication of mixed numbers may be performed in a shorter method, than by the general Rule, as follow:

CASE 1.

To multiply a *whole number* by a *fraction*.

Rule.

Multiply by the numerator of the fraction, and divide the product by the denominator.

Example.

Multiply 125 by $\frac{7}{8}$.

$\frac{7}{8}$

8)875

Ans. $109\frac{3}{8}$

6. Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $6\frac{1}{2}$ by $\frac{4}{5}$ of $\frac{1}{4}$ of $\frac{2}{7}$ of $6\frac{1}{2}$.
Ans. $6\frac{19}{175}$.

7. What is the continued product of $\frac{4}{11}$, 5, $4\frac{1}{4}$, and $\frac{3}{7}$ of $\frac{5}{9}$?
Ans. $1\frac{202}{693}$.

8. What is the continual product of $\frac{5}{8}$, 14, $31\frac{1}{2}$, and $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of 4?
Ans. $661\frac{1}{2}$.

CASE 2.

To multiply a *whole number* by a *mixed one.*

RULE.

Multiply by the fraction as in Case 1st. then multiply by the whole number, and add the products.

EXAMPLE.

Multiply 123
by $4\frac{3}{4}$

4)369

$92\frac{3}{4}$
492

$584\frac{1}{4}$ *the Product.*

CASE 3.

To multiply a *mixed number* by a *mixed number.*

RULE.

Multiply the integral part of the multiplicand by the denominator of its fractional part, and to the product add its numerator: then multiply by the mixed multiplier as in Case 2d. and divide the product by the denominator of the multiplicand.

DIVISION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as before; then invert the divisor, and proceed exactly as in multiplication.

EXAMPLES.

1. Divide $\frac{7}{10}$ by $\frac{4}{5}$.

$$\frac{7}{10} \div \frac{4}{5} = \frac{7}{10} \times \frac{5}{4} = \frac{7 \times 5}{10 \times 4} = \frac{35}{40} = \frac{7}{8} \text{ the Answer.}$$

2. Divide $4\frac{1}{10}$ by $\frac{9}{10}$. *Ans.* $4\frac{1}{9}$.
3. Divide $\frac{1}{4}$ by 4. *Ans.* $\frac{1}{16}$.
4. Divide $167\frac{2}{5}$ by $6\frac{3}{4}$. *Ans.* $24\frac{4}{27}$.
5. Divide $\frac{7}{8}$ by $13\frac{9}{10}$. *Ans.* $\frac{35}{351}$.

EXAMPLE.

Multiply $24\frac{4}{5}$ by $6\frac{3}{4}$ (*which mult. by*) $= 24\frac{4}{5} = 124$ (*which mult. by*) $6\frac{3}{4}$

4)372

93
744

5)837

$167\frac{3}{5}$ *Answer.*

Scholium. A fraction may be multiplied by an integer, by dividing the denominator by it. *Ex.* $\frac{3}{8} \times 2 = \frac{3}{4}$.

* A fraction may be divided by an integer, by dividing the numerator by it. *Ex.* $\frac{4}{12} \div 2 = \frac{2}{12} = \frac{1}{6}$ the quotient.

6. Divide $4\frac{12}{16}$ by $\frac{4}{7}$ of $\frac{7}{9}$. *Ans.* $7\frac{1}{4}$.
7. Divide $\frac{2}{4}$ of $\frac{1}{7}$ of 16 by $3\frac{1}{2}$. *Ans.* $\frac{24}{77}$.
8. Required the quotient of $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{4}{5}$ of $6\frac{1}{2}$ divided by $\frac{4}{5}$ of $\frac{1}{4}$ of $\frac{2}{3}$ of 2. *Ans.* $3\frac{1}{4}$.

DECIMAL FRACTIONS.

A *decimal* is a fraction, whose denominator is an unit, or 1, with as many cyphers annexed as the numerator has places; and is expressed by writing the numerator only, with a point before it called the separatrix.

Thus, ·5 is equal to $\frac{5}{10}$ or $\frac{1}{2}$.
·25 $\frac{25}{100}$ or $\frac{1}{4}$.
·75 $\frac{75}{100}$ or $\frac{3}{4}$.
$1\cdot7 = \frac{17}{10} = 1\frac{7}{10}$.
$\cdot04 = \frac{4}{100} = \frac{1}{25}$.
$\cdot005 = \frac{5}{1000} = \frac{1}{200}$.

A *finite* decimal is that, which ends at a certain place. But an *infinite* decimal is that, which has no end, or is indefinitely continued. An *infinite* or *repeating* decimal has one figure, or several, continually repeating; as ·66 66, &c. which is a *single repetend*; and ·363636, &c. which is a *compound repetend*. Repeating decimals are also called *circulates*, or *circulating decimals*. A point is placed over a single repetend; thus, $\dot{3}$; and a point over the first and last figures of a compound repetend; thus, $\dot{1}42857\dot{}$.

When other decimal figures precede a repetend, it is called a *mixed* repetend; as $\cdot1\dot{6}$ or $\cdot34\dot{7}6\dot{}$: otherwise, it is a *pure* or *simple* repetend; as $\cdot\dot{1}$ and $\cdot\dot{7}\dot{2}$.

The first place, next after the decimal point, is 10th. parts, the second is 100th. parts, &c. decreasing toward the right, or increasing toward the left in the same ratio, as integral or whole numbers do: Hence, we may consider unity to be a fixed point, from whence whole numbers proceed infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right, to nothing; as in the following

SCALE OF NOTATION.

C. Millions.	X. Millions.	Millions.	C. Thousands.	X. Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth parts.	Hundredth parts.	Thousandth parts.	X Thousandth parts.	C Thousandth parts.	Millionth parts.	X Millionth parts.	C Millionth parts.
9	8	7	6	5	4	3	2	1	·2	3	4	5	6	7	8	9

ADDITION OF DECIMALS.

Rule.

Place the numbers under one another, according to the value of their places: then find their sum as in whole numbers, and point off as many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Examples.

1. Add 26·25, 143·75, 106·125, 33·14689, 16·027, and 300 together.

26·25
143·75
106·125
33·14689
16·027
300

625·29889 *the Sum.*

2. Find the sum of 345, 30·067, 4·125, 48·75, 9·8523, and 400·56. *Ans.* 838·3543.
3. Required the sum of 12·8204, 400·125, 43·5, 9·0009, 160·38, and 20·123? *Ans.* 645·9493.
4. Required the sum of 84·302, 432·14, 8·895, 5000, 55·00087, and 48·3609. *Ans.* 5628·69877.

SUBTRACTION OF DECIMALS.

Rule.

Place the numbers according to their value; then subtract as in whole numbers, and point off the decimals as in addition.

Examples.

1. Find the difference of 1487·842 and 1306·428.

1487·842
1306·428

Difference 181·414

2. From 38·0678 take 8·75. *Ans.* 29·3178.
3. From 12345·25 take 5432·005. *Ans.* 6913·245.
4. From 800 take 123·456789. *Ans.* 676·543211.

MULTIPLICATION OF DECIMALS.

RULE.*

Place the factors and multiply as in whole numbers: then point off as many figures from the product as there are decimal places in both the factors; and if there be not so many places in the product, supply the defect by prefixing cyphers.

EXAMPLES.

1. Multiply 144
by ·125

720
1728

Product. 18·000

2. Multiply 8765 by ·7854. *Ans.* 6884·0310.
3. Multiply 4·123 by ·0321. *Ans.* ·1323483.
4. Multiply ·009876 by ·005432.
Ans. ·000053646432.

DIVISION OF DECIMALS.

RULE.†

Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor. If there be not so many places in the quotient as

---

* But any decimal may be multiplied by 10, 100, 1000, &c. by only removing the *separatux* in the multiplicand so many places to the right as there are cyphers in the multiplier. Ex. ·1728×10=1·728.

† But any decimal or mixed number may be divided by 10, 100, 1000, &c. by only removing the *separatrix* in the dividend so many places to the left as there are cyphers in the divisor. Ex. 1·728÷10=·1728.

the rule requires, supply the deficiency by prefixing cyphers. But if the decimal places in the divisor be more than those in the dividend, cyphers must be annexed to the dividend, or to the remainder, and the quotient may be carried on to any degree of exactness.

EXAMPLES.

1. Divide ·7854 by 14.

14)·7854(·0561.
70
85
84
14
14
..

2. Divide ·1323483 by ·0321. *Ans.* 4·123.
3. Divide 6884·031 by ·7854. *Ans.* 8765.
4. Divide 4567 by 1·25. *Ans.* 3653·6.
5. Divide ·000053646432 by ·005432. *Ans.* ·009876.

REDUCTION OF DECIMALS.

CASE 1.

To reduce a Vulgar Fraction to its equivalent decimal.

RULE.

Divide the numerator by the denominator, and the quotient will be the decimal required.

EXAMPLES.

1. Reduce $\frac{1}{16}$ to its equivalent decimal.

8)1·000
2)·125
·0625 *the Answer.*

2. Reduce $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to equivalent decimals.
Ans. ·25, ·3333, &c. ·5, ·6666, &c. *and* ·75.
3. Required the decimal expressions of $\frac{1}{8}$ and $\frac{4}{25}$.
Ans. ·375 *and* ·16.
4. Reduce $\frac{9}{125}$ of a dollar to the decimal of a pound.
Ans. ·0216.
*5. Reduce 15s. $6\frac{3}{4}$d to the decimal of a pound.
Ans. L. ·778125.
6. Reduce *L.* 3. 18s. $2\frac{2}{7}$d. to the decimal of *L.* 8. 16s.
Ans. ·444, &c.
7. Reduce *L.* 1. 2s. $6\frac{1}{8}$d. to the decimal of a guinea.
Ans. ·8046875.
8. Reduce. *L.* 1. 8s. 9d. $2\frac{2}{5}$qr. to the decimal of a moidore. *Ans.* ·8.
9. Reduce *L.* 1. 17s. 6d. to the decimal of an eagle.
Ans. ·625.
10. Reduce 10oz. 10pwt. to the decimal of a lb troy.
Ans. ·875.
11. Reduce $2\frac{3}{4}$. $2\frac{3}{4}$. to the decimal of a lb. apothecaries'.
Ans. ·1875.
12. Reduce 6Cwt. 0qr. 17lb. 3oz. $11\frac{1}{7}$dr. to the decimal of a ton. *Ans.* ·307692, &c.
13. Reduce 6m. 1fur. 32r. to the decimal of $41\frac{1}{2}$ miles.
Ans. ·15.
14. Reduce 1qr. $3\frac{1}{2}$na. to the decimal of an English ell.
Ans. ·375.
15. Reduce 3 roods and 24 poles to the decimal of an acre.
Ans. ·9.
16. Reduce 33ft. 1152in. of hewn timber to the decimal of a ton.
Ans. ·666, &c.
17. Reduce $28\frac{1}{4}$ galls. of wine to the decimal of a hhd.
Ans. ·4583, &c.
18. Reduce 12 gall. $3\frac{1}{5}$ qts. of ale to the decimal of a hhd.
Ans. ·26, &c.

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*To do which, reduce 15*s.* $6\frac{3}{4}$*d.* to the fraction of a pound by *Case* 10*th.* in *Reduction of Vulgar Fractions*, and it becomes *L.* $\frac{240}{320}$; which reduced to a decimal, gives, *L.* ·778125 *the Answer as above.*

19. Reduce $3\frac{1}{3}$ bushels of corn to the decimal of a quarter. Ans. ·41[illegible], &c.

20. Reduce 344 d. 23 h. to the decimal of a Julian year. ·94, &c.

21. Reduce 18' 45" to the decimal of [illegible]. ·3125.

CASE 2.

To find the value of any given decimal in the terms of the integer.

RULE.

Multiply the decimal by the common parts of the integer.

EXAMPLES.

1. Find the value of ·778125 of a pound.

·778125
20
15·562500
12
6·750000
4
3·000000

Answer 15*s.* $6\frac{3}{4}$*d.*

2. Find the value of *L.* ·755. Ans. 15*s.* $1\frac{1}{5}$*d.*
3. What is the value of ·25 of a pistole?
Ans. 5*s.* 6*d.*
4. What is the value of ·8046875 of a guinea?
Ans. *L.* 1 2*s.* 6·375*d.*
5. What is the value of ·8 of a moidore?
Ans. *L.* 1 8*s.* 9*d.* 2·4*qr.*
6. What is the value of ·5625 of a half joe?
Ans. *L.* 1 7*s.*
7. Find the value of ·625 of an eagle.
Ans. *L.* 1 17*s.* 6*d.*

8. Find the value of ·875 [illegible] lb. troy.
Ans. 10 *oz.* 10 *pwt.*

9. What is the value of ·1875 of a lb apothecaries ?
Ans. 2$\bar{3}$. 23.

10. What is the value of ·32 of a ton avoirdupois ?
Ans. 1 *hhd.* 17 *gal.* [illegible] *pint.*

11. What is the value of ·15 of [illegible] 1*fur.* 32*r.*

12. What is the [illegible] English ell ?
Ans. 1*qr.* 3$\frac{1}{2}$*na.*

13. The value of ·9 of an acre is required.
Ans. 3*r.* 24 *pol.*

14. What is the value of ·025 of a cord of wood ?
Ans. 3·2*ft.*

15. What is the value of ·9375 of a tun of wine ?
Ans. 3 *hhd.* 47 *gal.* 1 *qt.*

16. What is the value of ·61 of a *hhd.* of beer ?
Ans. 32 *gal.* 3 *qts.* 1·52 *pt.*

17. What is the value of ·92 of a chaldron of coals ?
Ans. 33$\frac{1}{2}\frac{1}{5}$ *bushels.*

18. What is the value of ·85714 of a month ?
Ans. 3*w.* 2*d.* 23*h.* 59*m.* 56*sec.*

19. What is the value of ·3125 of a degree ?
Ans. 18′ 45″.

FEDERAL MONEY.*

The denominations of *Federal Money* are in a *decuple* or *decimal ratio ;* and, therefore, may be properly introduced in this place.

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* The federal coins are *eight ;* two of gold, four of silver, and two of copper. The gold coins are called an *eagle* and *half eagle ;* the silver, a *dollar, half dollar, double dime,* and *dime ;* and the copper, *cent* and *half*

A mill, which is the lowest money of account, is ·001 of a dollar, which is the money unit.

A cent	is	·01
[illegible]dime		·1
[illegible]ar		1·
[illegible]gle		10·

Or, 10 mills = 1 cent, *marked* m. c.
10 cents = 1 dime, ... d.
10 dimes = 1 dollar, ... D.
10 dollars = 1 eagle, ... E.

EXAMPLES.

1. Add $24\frac{1}{4}$ eagles; 8 dollars, 7 dimes, 4 cents, 8 mills; 425 dollars, 9 cents; 8 eagles, 4 mills; 200 dollars; $25\frac{1}{4}$ dollars; 4 eagles, and $3\frac{1}{2}$ dollars together.*

---

cent. The standard for gold and silver is eleven parts fine and one part alloy. The weight of fine gold in the eagle is 246 ·268 grains; of the half eagle, 123 ·134 grains; of fine silver in the dollar, 375 ·64 grains; of the half dollar, 187 ·82 grains; of the double dime, 75 ·128 grains; of the dime, 37 ·564 grains; and of copper in 100 cents, $2\frac{1}{4}$ *lb*. avoirdupois. The mint-price of uncoined, standard gold is 209 ·77 dollars per pound troy weight; and the mint-price of uncoined, standard silver, 9 ·92 dollars a pound troy. When either gold or silver is finer or coarser than standard, the variation from standard is estimated by carats and grains of a carat in gold, and by pennyweights in silver.

Note 1. *Alloy* is used in gold or silver to harden them.

2. A *carat* is not any certain weight or quantity, but $\frac{1}{24}$ of any weight or quantity; and the minters and goldsmiths divide it into 4 equal parts called grains of a carat.

* It may be observed, that dollars occupy the first place on the left hand of the *separatrix*, and eagles, all

Eagles.	*Dollars.*	*dimes.*	*cents.*	*mills.*
2	4	2	5	
		8	7	4 8
4	2	5	0	9
		8	0	0 4
2	0	0		
		2	5	7 5
		4	3	5
1	0	2	5	5 9 2 *Sum.*

2. Add 285 dollars; 554 mills; 45 eagles, 68 cents; 350 dollars, 8 mills; $44\frac{1}{2}$ dollars; $10\frac{1}{8}$ dollars; and $25\frac{1}{4}$ eagles together. Ans. 1398·367.

3. Add 1000 D. $100\frac{1}{4}$ D. $8\frac{1}{4}$ d. $15\frac{1}{2}$ c. $25\frac{1}{8}$ D. 400 D. and $9\frac{1}{4}$ d. together. Ans. 1527·83.

4. The sum of 245·6 D. $30\frac{1}{4}$ D. 2400 D. 100·975 D. 9·005 D. ·048 D. and 800 D. is required.

Ans. 3585·878.

5. Find the difference between 874·845 dollars and 560·908 dollars.

E.	*D.*	*d.*	*c.*	*m.*
8	7	4·	8	4 5
5	6	0·	9	0 8
Answer 3	1	3·	9	3 7

---

the places on the left of dollars; but eagles and dollars reckoned together, express the number of dollars contained in the sum; as, 1025 is 102 Eagles and 5 Dollars; equal to 1025 Dollars, &c.

It may be farther observed, that the sum exhibits the particular number of each different piece of money contained in it, *viz.* 1025592 *mills* $=$ $102559\frac{2}{10}$ *cents* $=$ $10255\frac{92}{100}$ *dimes* $=$ $1025\frac{592}{1000}$ *Dollars* $=$ $102\frac{5592}{10000}$ *Eagles* $=$ 102 *E.* 5 *D.* 5 *d.* 9 *c.* 2 *m.*

6. From 1250 D. 6 m. take 183 D. 12½ c.
Ans. 1066 D. 8 d. 8 c. 1 m.

7. From 467·25 D. take 134·666 D.
Ans. 332·584 D.

8. From 1000000 D. take 123456·789 D.
Ans. 876543·211 D.

9. Multiply 12·125 D. d. c. m. by 3 d. 2 c. 1 m.
·321

12125
24250
36375

3·892125 = 3 D. 8 d. 9 c. 2 m. 125/1000 *Ans.**

10. Multiply 206·25 D. by 432. Ans. D. 89100.

11. Multiply 44·006 D. by 25·907 D.
Ans. 1140.063442 D.

12. Multiply 80·875 D. by 40·072 D.
Ans. 3240·823 D.

13. Divide 3·892125 D. by ·321 D.

·321)3·892125(12·125 D. Ans.
3 21

682
642

401
321

802
642

1605
1605

....

* The figures on the right hand of *mills* are *decimals* of a mill.

14. Divide 89100 D. by 432. Ans. 206·25 D.
15. Divide 1058·767524 D. by 23·094 D.
Ans. 45·846 D.
16. Divide 3240·823 D. by 40·072 D.
Ans. 80·875 D.

CIRCULATING DECIMALS.

Circulating Decimals are produced from Vulgar Fractions whose denominators do not measure their numerators.

Similar repetends begin at the same place; as $\cdot\dot{3}$ and $\cdot\dot{6}$, or $\cdot\dot{1}\dot{6}$ and $\cdot\dot{8}\dot{3}$.

Dissimilar repetends begin at different places; as $\cdot 1\dot{8}$ and $\cdot 0\dot{8}\dot{3}$.

Conterminous repetends end at the same place; as $\cdot\dot{8}\dot{1}$ and $\cdot\dot{9}\dot{0}$.

Similar and *conterminous repetends* begin and end at the same place; as $3\cdot\dot{1}4285\dot{7}$ and $6\cdot\dot{2}8571\dot{4}$.

Scholium. Any *finite* decimal may be considered as *infinite*, by annexing cyphers continually to the right hand of the numerator; thus, $\cdot 31 = 31000$, &c. $= .340$. And any *pure* circulate may be considered as *mixed*, by taking the given repetend for a *finite* part, and making the same repetend a circulate for the *infinite* part; thus, $\cdot\dot{3}\dot{1} = \cdot 31 + \cdot 0031$.

REDUCTION OF CIRCULATING DECIMALS.

CASE 1.

To reduce a simple repetend to its equivalent Vulgar Fraction.

RULE.

1. Make the given decimal the numerator, and let the denominator be a number consisting of as many nines as there are recurring places in the repetend.

2. If there be ietegral figures in the circulate, as many cyphers must be annexed to the numerator as the highest place of the repetend is distant from the decimal point.

EXAMPLES.

1. Required the least vulgar fractions equal to $\cdot\dot{6}$ and $\cdot\dot{3}\dot{6}\dot{9}$.

$\cdot\dot{6} = \frac{6}{9} = \frac{2}{3}$; *and* $\cdot\dot{3}\dot{6}\dot{9} = \frac{369}{999} = \frac{41}{111}$ Ans. $\frac{2}{3}$ *and* $\frac{41}{111}$.

2. Reduce $\cdot\dot{9}\dot{0}$ to its equivalent vulgar fraction. Ans. $\frac{10}{11}$.

3. Reduce $3\cdot\dot{5}0649$ to its equivalent vulgar fraction. Ans. $\frac{3506400}{999999}$.*

4. Required the least vulgar fraction equal to $\cdot\dot{4}6153\dot{8}$. Ans. $\frac{6}{11}$.

CASE 2.

To reduce a mixed repetend to its equivalent vulgar fraction.

RULE.

1. To as many nines as there are figures in the repe-

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* For $3\cdot\dot{5}0649 = 3\cdot\dot{5}06493 = 3\frac{506491}{999999} = 3\frac{506400}{999999} = 3\frac{270}{77} = 3\frac{3}{77}$.

tend, as many cyphers as there are finite places, for a denominator.

2. Multiply the nines in the said denominator by the finite part, and add the repeating decimal to the product for the numerator.

3. If the repetend begin in some integral place, the finite value of the circulating part must be added to the finite part.

EXAMPLES.

1. What is the vulgar fraction equivalent to $\cdot\dot{2}\dot{7}$?

$9\times2+7=25=$ *numerator, and* 90 *the denominator;* $\therefore\ \cdot\dot{2}\dot{7}=\frac{25}{90}=\frac{5}{18}$ *the Answer.*

2. What is the least vulgar fraction equivalent to $\cdot\dot{5}9259\dot{}$? Ans. $\frac{16}{27}$.

3. What is the least vulgar fraction equivalent to $\cdot\dot{8}57142\dot{8}57\dot{}$? Ans. $\frac{6}{7}$.

4. What is the finite number equivalent to $41\cdot\dot{6}2$? Ans. $41\frac{2}{3}$.

CASE 3.

To make any number of dissimilar repetends similar and conterminous.

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* In this *Example*, you must find the finite value of the circulating part by Case 1st. and add it to the finite part. Or you may, according to the *Scholium*, divide the circulating part into finite and infinite parts; then find, in its lowest terms, the finite value of the mixed repetend and add it to the integral part; and the sum will be the answer as above.

Rule.

Change them into other repetends, which shall each consist of as many figures as the least common multiple of the several numbers of places, found in all the repetends, contains units.

Examples.

1. Make $\cdot\dot{0}8\dot{3}$; $4\cdot\dot{3}\dot{6}$; $\cdot\dot{0}9\dot{0}\dot{2}$; and $9\cdot\dot{8}1\dot{4}$ similar and conterminous.

In the first place, find the least common multiple of the several numbers of the circulates, and it is 6 units; therefore, the similar and conterminous repetends must contain 6 places.*

Dissimilar		*Made similar and conterminous.*
$\cdot\dot{0}8\dot{3}$	═══	$\cdot08333333$
$4\cdot\dot{3}\dot{6}$	═══	$4\cdot36363636$
$\cdot\dot{0}9\dot{0}\dot{2}$	═══	$\cdot09029029$
$9\cdot\dot{8}1\dot{3}$	═══	$9\cdot81481481$

2. Make $12\cdot\dot{0}9$, $1\cdot\dot{5}$, $3\cdot\dot{2}\dot{6}$, and $\cdot\dot{7}3\dot{1}\dot{8}$ similar and conterminous.

3. Make $\cdot\dot{3}$, $\cdot0\dot{5}0\dot{3}$, $24\cdot\dot{2}\dot{5}$, and $\cdot0\dot{4}\dot{5}$ similar and conterminous.

4. Make $16\cdot285\dot{7}1\dot{4}$, $6\cdot\dot{8}\dot{3}$, $\cdot\dot{1}1\dot{3}\dot{6}$, and $4\cdot\dot{3}6\dot{7}\dot{1}\dot{2}$ similar and conterminous.

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* The learner may observe that the similar and conterminous repetends begin just as far from unity, as is the farthest among the dissimilar repetends; and is so in all cases.

CASE 4.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and of how many places the repetend will consist.

RULE.

1. Reduce the given fraction to its least terms, and divide the denominator by 2, 5 or 10, as often as possible.
2. If the whole denominator vanish in dividing by 2, 5 or 10, the decimal will be finite, and will consist of so many places as you perform divisions.
3. If it do not so vanish, divide 9999, &c. by the result, till nothing remain, and the number of 9's used will shew the number of places in the repetend; which will begin after so many places of figures, as there were 10's, 2's or 5's, used in dividing.

EXAMPLES.

1. Required to find whether the decimal equal to $\frac{120}{1050}$ be finite or infinite, and if infinite, of how many places will the repetend consist.

First $\frac{120}{1050}=\frac{5}{44}$, 2)44 | 22 | 11 ; *Then* 11)99 ; *and* 09

therefore the decimal is infinite, and the circulate consists of 2 *places, beginning at the third place from the decimal point.*

2. Required to find whether the decimal equal to $\frac{21}{111}$ be finite or infinite; and, if infinite, of how many places will the circulate consist.

3. Let $\frac{5}{11}$ be the fraction proposed.
4. Let $\frac{1}{7}$ be the fraction proposed.
5. Let $\frac{1}{13}$ be the fraction proposed.
6. Let $\frac{1}{56}$ be the fraction proposed.
7. Let $\frac{1}{208}$ be the fraction proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE.

1. Make the repetends similar and conterminous, and find their sum as in common addition.

2. Divide this sum by as many nines as there are places in the repetend, and the remainder is the repetend of the sum; which must be set under the figures added, with cyphers on the left hand, when it has not so many places as the repetends.

3. Carry the quotient of this division to the next column, and proceed with the rest as in finite decimals.

EXAMPLES.

1. Let $3\cdot\dot{6}$, 360, $4\cdot3\dot{4}76$, $10\cdot5$, $24\cdot0\dot{8}3$, and $6\cdot75$ be added together.

Dissimilar.		*Similar and conterminous.*
$3\cdot\dot{6}$	═══	$3\cdot\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}$
360	═══	$360\cdot\dot{0}\dot{0}\dot{0}\dot{0}\dot{0}$
$4\cdot3\dot{4}76$	═══	$4\cdot3\dot{4}76\dot{4}$
$10\cdot5$	═══	$10\cdot5\dot{0}\dot{0}\dot{0}\dot{0}$
$24\cdot0\dot{8}3$	═══	$24\cdot0\dot{8}3\dot{3}\dot{3}$
$6\cdot75$	═══	$6\cdot75\dot{0}\dot{0}\dot{0}$
		$409\cdot3\dot{4}76\dot{4}$ *the Sum.**

* In this *Example*, the sum of the repetends is 1763, which divided by 999 gives 1, to carry, and the remainder is 764.

2. Add $6\cdot\dot{3}$, $2\cdot\dot{2}\dot{7}$, 240, $12\cdot\dot{1}\dot{6}$, $2\cdot\dot{6}$, and $3\cdot0\dot{3}$ together.

Ans. $266\cdot4\dot{0}\dot{6}$.

3. Add $2\cdot\dot{2}5$, $3\cdot\dot{3}$, 100, $4\cdot4\dot{3}5\dot{6}$, $85\cdot0\dot{1}\dot{6}$, and $50\cdot\dot{5}$. together. Ans. $245\cdot5\dot{3}\dot{5}\dot{6}\dot{3}$.

4. Add $13\cdot\dot{3}07692$, $15\cdot\dot{0}6$, 400, $132\cdot7\dot{3}\dot{4}8$, $264\cdot\dot{3}6\dot{7}\dot{4}\dot{2}$, and $8\cdot\dot{1}25$ together. Ans. $833\cdot601631\dot{7}0\dot{1}$.

SUBTRACTION OF CIRCULATING DECIMALS.

RULE.

Make the repetends similar and conterminous, and subtract as usual; then, if the repetend of the subtrahend be greater than the repetend of the minuend, the right hand figure of the remainder must be less by unity, than it would be, if the expressions were finite.

EXAMPLES.

1. From $150\cdot0\dot{3}$ take $100\cdot\dot{0}1\dot{6}$

$150\cdot0\dot{3} = 150\cdot0\dot{3}\dot{3}$

$100\cdot\dot{0}1\dot{6} = 100\cdot\dot{0}1\dot{6}$

$50\cdot\dot{0}1\dot{6}$ *the Difference.*

2. From $1234\cdot\dot{2}7$ take $1000\cdot\dot{3}2$. Ans. $233\cdot9\dot{5}0$.

3. From 400·037 take 200·3476. Ans. 199·6893.

4. From 45·63 take 34·76432. Ans. 10·87203.

MULTIPLICATION OF CIRCULATING DECIMALS.

Rule.

1. Reduce both the terms to their equivalent vulgar fractions, and find the product of those fractions as usual.

2. Reduce the vulgar fraction, expressing the product, to an equivalent decimal, and it will be the product required.

Examples.

1. Multiply ·36 by ·25.

$\cdot 36 = \frac{36}{99} = \frac{12}{33} = \frac{4}{11}$ and $\cdot 25 = \frac{25}{99}$; $\therefore$

$\frac{4}{11} \times \frac{25}{99} = \frac{92}{990} = \frac{46}{495} = \cdot 092$ *the Product.*

2. Multiply 32·5 by ·81. Ans. 26·590.

3. Multiply 20·769230 by 4·3. Ans. 90.

4. Multiply 8·4356 by 4·1136. Ans. 34·7011375.

DIVISION OF CIRCULATING DECIMALS.

Rule.

1. Reduce both the divisor and dividend to their equivalent vulgar fractions, and find their quotient as usual.

2. Reduce the vulgar fraction, expressing the quotient, to its equivalent decimal, and it will be the quotient required.

EXAMPLES.

1. Divide ·092 by ·25.

$$\cdot 092 = \tfrac{92}{990} = \tfrac{46}{495} \text{ and } \cdot 25 = \tfrac{21}{90}; \quad \therefore$$

$$\tfrac{46}{495} \div \tfrac{21}{90} = \tfrac{46}{495} \times \tfrac{90}{21} = \tfrac{4140}{11385} = \tfrac{4}{11} = \cdot 36 \textit{ the quotient.}$$

2. Divide 26·590 by ·81. Ans. 32·5.

3. Divide 90 by 4·3. Ans. 20·769230.

4. Divide 31·7011375 by 4·1136. Ans. 8·4356.

DUODECIMALS;

or,

CROSS MULTIPLICATION.

Duodecimals or Cross Multiplication is a Rule which is sometimes used in finding the contents of superficies and solids.

Dimensions are generally taken in feet, inches, and parts.

Inches and parts are sometimes called primes, seconds, thirds, fourths, &c. and are marked thus; primes or inches (′), seconds (″), thirds (‴), fourths (⁗), &c.

This method of multiplying, however, is not confined to twelves, but may be extended at pleasure; for any number, whether its inferior denominations decrease from

an integer in the same ratio, or not, may be multiplied by this Rule, by observing, that, if any denomination be multiplied by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand, but, if multiplied by any number of an inferior denomination, the value of an unit in the product will be so much inferior to the value of an unit in the multiplicand, as an unit in the multiplier is less than an integer; therefore,

RULE.

Under the multiplicand write the corresponding denominations of the multiplier: multiply every denomination in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each under its respective denomination, and carry from one denomination to another according as an unit in the number by which you multiply is equal or less than an integer; in the same manner, multiply the multiplicand by the next inferior denomination in the multiplier, and thus proceed by every term; and the sum of the products will be the answer required.

EXAMPLES.

1. Multiply $3\frac{1}{2}$ feet by $3\frac{1}{2}$ feet.

	F.	'		*Or thus.*
	3	6		$3\frac{1}{2}$
	3	6		$3\frac{1}{2}$
	10	6	"	$10\frac{1}{2}$
	1	9	0	$1\frac{1}{4}$
Ans.	12	3	0	*Ans.* $12\frac{1}{4}$ *square feet* = [12 *ft.* 36 *in.*

So that 3 is not 3 inches, but 36 inches, or $\frac{1}{4}$ of a square foot.

2. Multiply 14 ft. 9 inches by 4 ft. 6 inches.

Ans. 66 *ft.* 4'. 6".

3. Multiply 4 ft. 3′. 6″. by 2 ft. 6′. 9″.
Ans. 10 *ft.* 11′. 11″. 7‴. 6⁗.

4. Multiply 8 ft. 4′. 7″. by 8′. 5″. 4‴.
Ans. 5 *ft.* 10′. 9″. 4‴. 5⁗. 4⁗′.

5. How many square feet in a board which is 19 feet 7 inches long, and 1 foot 5 inches wide?
Ans. 27 *ft.* 8′. 11″.

6. How many solid feet in a stick of timber 14 feet 11 inches long, 1 foot 9 inches wide, and 1 foot 5 inches thick?
Ans. 36 *ft.* 11′. 9″. 3‴.

7. How many cubic feet of wood in a load 7 feet 7 inches long, 3 feet 5 inches wide, and 3 feet 8 inches high?
Ans. 95 *ft.* 0′. 0″. 4‴.

8. What is the product of 2*s.* 6*d.* by 2*s.* 6*d.*?
Ans. 6*s.* 3*d.*

9. What is the rectangle of *L.* 2 3*s.* 9*d.* by *L.* 2 1*s.* 6*d.*?
Ans. L. 4 10*s.* $9\frac{3}{8}$*d.*

OF PROPORTION IN GENERAL.

Numbers are compared together to discover the relations which they have to each other.

There must be two numbers to form a comparison: the number, which is compared, being written first, is called the *antecedent;* and that, to which it is compared, the *consequent.*

Numbers are compared with each other two different ways: one comparison considers the *difference* of the two numbers, and is called *arithmetical relation,* the difference is sometimes named the *arithmetical ratio;* and the other considers their *quotient,* and is termed *geometrical relation,* and the quotient, the *geometrical ratio.**

* Ratios are, here, considered as the result of the greater term of comparison diminished, or divided, by the less; regarding not which of them be the antecedent.

If two or more couplets of numbers have equal ratios, the equality is named *proportion ;* and their terms similarly posited, i. e. either all the greater, or all the less, taken as antecedents, and the rest as consequents, are called *proportionals.* So the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8 ; or thus, 4, 2, 8, 6, are arithmetical proportionals ; and the two couplets 2, 4, and 8, 16, taken thus 2, 4, 8, 16, or thus 4, 2, 16, 8, are geometrical proportionals.*

Proportion is distributed into *continued* and *discontinued.*

If, of several couplets of proportionals written down in a series, the ratio of each consequent and the antecedent of the next following couplet be the same as the common ratio of the couplets, the *proportion* is said to be *continued*, and the numbers a series of *continued arithmetical* or *geometrical proportionals.* Thus 2, 4, 6, 8, form an arithmetical progression ; for $4-2=6-4=8-6=2$; and 2, 4, 8, 16, a geometrical progression ; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$.

But if the ratio of the consequent of one couplet, and the antecedent of the next, be not the same as the common ratio of the couplets, the proportion is said to be *discontinued.* So 4, 2, 8, 6, are in *discontinued arithmetical proportion ;* for $4-2=8-6=2$; but $8-2=6$; and 4, 2, 16, 8, are in *discontinued geometrical proportion ;* for $\frac{4}{2}=\frac{16}{8}=2$; but $\frac{16}{2}=8$.

Four numbers are *directly porportional*, when the ratio of the first to the second is the same as that of the

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* In geometrical proportionals, a colon is written between the terms of each couplet, and a double colon, between the couplets ; in arithmetical proportionals, a colon is turned horizontally between the terms of each couplet, and two colons placed between the couplets. Hence, the above geometrical proportionals are written thus, 2 : 4 :: 8 : 16, and 4 : 2 :: 16 : 8 ; the arithmetical proportionals, thus, 2 ·· 4 :: 6 ·· 8, and 4 ·· 2 :: 8 ·· 6.

third to the fourth. As 2 : 4 : : 5 : 10. Four numbers are said to be *reciprocally* or *inversely proportional*, when the first is to the third, as the fourth is to the second, and the contrary. Thus, 2, 9, 6, 3, are reciprocal proportionals ; 2 : 6 : : 3 : 9, or 6 : 2 : : 9 : 3.

Four numbers are said to be in *harmonical proportion*, when the difference of the first and second is to the difference of the third and fourth as the first is to the fourth: thus, 3, 4, 6, 9, are harmonical proportionals ; for 4—3=1 : 9—6=3 : : 3 : 9.

THEOREM 1.

If any four quantities be in arithmetical proportion ; the sum of the extremes is equal to the sum of the means.* Therefore, of three arithmetical proportionals, the sum of the extremes is double the mean.

THEOREM 2.

If four quantities be in geometrical proportion ; the product of the extremes is equal to the product of the means.†

Corollary 1. Hence, of three geometrical proportionals, the product of the extremes is equal to the square of the mean.

Corollary 2. From hence it is manifest, that either extreme of four geometrical proportionals is equal to the product of the means divided by the other extreme ; and that either mean is equal to the product of the extremes divided by the other mean.

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* Thus of 2 ··4 : : 6 .. 8, the sum of the extremes (2+8)=the sum of the means (4+6)=10.

† Thus, of 2 : 4 : : 8 : 16, the product of the extremes (2×16)=the product of the means (4×8)=32.

SIMPLE PROPORTION.*

Simple Proportion or the Rule of Three is that, by which a number is found, having to a given number the same ratio, that is between two other given numbers.

RULE.†

Write down the number, which is of the same kind with the number or answer required; then, consider whether the answer should be greater or less than this number; if *greater*, write the *greater* of the two remaining numbers on the left hand of it for the *second number* or *term*, and the other for the *first*; but if *less*, write the *less* of the two remaining numbers in the *second*, and the other in the *first place*. The question being thus stated the *proportion* will be; as the first number or term is to the second; so is the third to the fourth, or answer required.

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* Simple Proportion, on account of its great and extenfive uſefulneſs, is ſometimes called the *golden Rule* of *Proportion*: for, on a proper application of it and the preceding rules, the whole of Arithmetic as well as every mathematical enquiry depends.

† Beſides this general Rule, there are four others, any of which, when applicable, performs the work more concifely. They are as follow.

RULE 1.

Divide the ſecond term by the firſt, multiply the quotient into the third, and the product will be the anſwer.

RULE 2.

Divide the third term by the firſt, multiply the quotient by the ſecond, and the product will be the anſwer.

RULE 3.

Divide the firſt term by the ſecond, the third by the quotient, and the laſt quotient will be the anſwer.

Therefore, Multiply the second and third terms together, divide the product by the first, and the quotient will be the answer.

NOTE 1. It is sometimes most convenient to multiply and divide as in compound multiplication and division; and sometimes it is expedient to multiply and divide according to the rules of vulgar or decimal fractions. But when neither of these methods is chosen, reduce the compound terms, each to the lowest denomination expressed in it, and the first and second to the same denomination; then will the answer be of the same denomination with the third term. And the answer may afterwards be reduced to any denomination required.

RULE 4.

Divide the first term by the the third, divide the second by the quotient, and the last quotient will be the answer.

The general Rule above given is tantamount to those, which are commonly given in the direct and inverse rules of three, and which are here subjoined.

The Rule of Three direct teaches from three numbers given to find a fourth, that shall have the same proportion to the third, as the second has to the first.

RULE 1.

State the question by placing the numbers in such order, that the first and third may be the terms of supposition and demand, and the second of the same kind with the answer.

RULE 2.

Reduce the first and third terms to the same name or denomination, and the second to the lowest mentioned.

RULE 3.

Multiply the second and third numbers together, divide the product by the first, and the quotient will be the answer, in the same denomination in which you left the second number; which may be brought into any other denomination required.

NOTE 2. If the first term and either the second or third can be measured by any number, let them be divided, and the quotients used instead of them.

NOTE 3. *Direct* and *Inverse* proportion are only parts of the same general rule, and are, therefore, contained in the preceding.

EXAMPLES.

If 24lb. of raisins cost 6s. 9d. what will 3 Cwt cost?

lb.	*s.*	*d.*			
If 24	: 6	9	: :	3 Cwt.	
12				4	
81				12	
				7	
				84	
				4	
				336	
				81	
				336	
				2688	
				4)27216	
				6)6804	
				12)1134	
				2,0)9,4	6

L. 4. 14*s.* 6*d.* *Answer.*

The Rule of Three inverse is that, which teaches, by having three numbers given, to find a fourth, that shall have the same proportion to the second as the first has to the third.

Therefore, the greater the third term is, in proportion to the first, the less will the fourth term be in proportion to the second; or, the less the third term is in respect to the first, the greater the fourth term must be in proportion to the second; and this is termed reciprocal, indirect, or inverse proportion.

RULE 1.

State and reduce the terms as in the rule of three direct.

NOTE 4. Two or more statings are sometimes necessary, which may always be known from the nature of the question.

NOTE 5. The method of proof is by reversing the question.

RULE 2.

Multiply the first and second terms together, and divide their product by the third, and the quotient is the answer to the question, in the same denomination to which the second term was reduced.

Note 1. If *more* require *more*, or *less* require *less*, the question belongs to the rule of three direct.

But if *more* require *less*, or *less* require *more*, it belongs to the rule of three inverse.

Note 2. The meaning of these phrases "If *more* require *more*, *less* require *less*," &c. must be understood thus: *more* requires *more*, when the third term is greater than the first, and requires the fourth to be greater than the second; *less* requires *less*, when the third term is less than the first, and requires the fourth to be less than the second; *more* requires *less*, when the third term is greater than the first, and requires the fourth to be less than the second; and *less* requires *more*, when the third term is less than the first, and requires the fourth to be greater than the second.

Note 3. The method of proof, whether the proportion be direct or inverse, is by inverting the question.

EXAMPLE.

If 100 men can build a wall in 27 days, how many men can build the same in 90 days?

d. *m.* *d.*
If 27 : 100 : : 90
100

9,0)270,0

30 *men, Answer.*

Now, to prove this, reverse the question and work as before; thus,

If 30 men can build a wall in 90 days, in what time can 100 men perform it?

m. *d.* *m.*
If 30 : 90 : : 100
30

1,00)27,00

Ans. 27 *Proof.*

EXAMPLES.

1. If $1\frac{1}{2}$ yard of linen cost 7s. 6d. what will $48\frac{1}{4}$ yards cost at that rate?

yd.	*yds.*	*s.*	*d.*
$1\frac{1}{2}$:	$48\frac{1}{4}$: :	7	6 :
4	4	12	
6	195	90	

6)17550
12)2925
2,0)24,3·9

L. 12 3*s.* 9*d.* *Ans.*

The same by Vulgar Fractions.

First $1\frac{1}{2}=\frac{3}{2}$. 7s. 6d.=L. $\frac{3}{8}$, *and* $48\frac{1}{4}=\frac{195}{4}$, *then*

yd.	*yd.*	*L.*
$\frac{3}{2}$:	$\frac{195}{4}$: :	$\frac{3}{8}$:

And $\frac{195}{4}\times\frac{3}{8}\div\frac{3}{2}=\frac{195}{4}\times\frac{3}{8}\times\frac{2}{3}=\frac{195\times3\times2}{4\times8\times3}$=L. $\frac{117}{96}$=

L. 12. 3*s.* 9*d. the Answer as before.*

The same by Decimal Fractions.

$1\frac{1}{2}=1\cdot5$, 7s. 6d.=L.·375, *and* $48\frac{1}{4}=48\cdot75$, *then*

yds.	*yds.*	*L.*
1·5 :	48·75 : :	·375 :

·375
24375
34125
14625

L. *L.* *s.* *d.*

1.5)18·28125(12·1875=12 3 9 *as above.*

15
32
30
28
15
131
120
112
105
75
75

Or thus, If 30 men can build a wall in 90 days, how may men can accomplish the same in 27 days?

2. If 1 *cwt.* of tobacco cost *L.* 5 13*s.* 4½*d.* what will 9 *cwt.* cost? *Ans. L.* 51 0*s.* 4½*d.*

3. If 1 pair of silk stockings cost 19*s.* 6¾*d.* what will 10 dozen pair cost? *Ans. L.* 117 7*s.* 6*d.*

4. If 36 yards of broadcloth cost *L.* 58 10*s.* what is that a yard?

Ans. L. 1 12*s.* 6*d.*

5. What will a *Hhd.* of rum come to at 12½ cents a pint? *Ans.* 63 *Dollars.*

6. Bought 10 yards of superfine broadcloth for 48·75 dollars; what did it cost a yard?

Ans. 4·875 *dollars.*

7. Bought 10 chests of tea, each weighing 2 *cwt.* 3 *qrs.* 25 *lb.* for *L.* 136 15*s.* 4¾*d.* what did it cost me a *Cwt.*? *Ans. L.* 4 12*s.*

8. A merchant has owing to him 4000 dollars, and his debtor agrees to pay him for every dollar 64 cents; how much must he pay in all?

Ans. 2560 *dollars.*

9. If a Gentleman's income be 200 guineas annually, I desire to know what he may spend a day so that he may lay up *L.* 80 a year. *Ans.* 10*s.* 11⅓⅞*d.*

10. A man bought a piece of cloth for 60 dollars, at 1½ dollar a yard; how many yards were there in the piece? *Ans.* 40 *yds.*

11. If a ship cost 6721 dollars; what are ⅝ of her worth? *Ans.* 4200 *dollars* and 62½ *cents.*

---

d. *m.* *d.*
If 90 : 30 : : 27
30

27)2700(100 *men, Answer.*

12. If 100 workmen can finish a piece of work in 24 days; how many are sufficient to do the same in 3 days? *Ans.* 800 *men.*

13. A merchant, failing in trade, owes in all *L.*6950, and has in money and effects but *L.*4633 6*s.* 8*d.*; now, supposing his money and effects are delivered up, what will each creditor receive on the pound?

Ans. 13*s.* 4*d.*

14. If ·375 of a yard of silk velvet cost 1·5 dollar; how many yards can I buy for 100 dollars?

Ans. 25 *yards.*

15. How much in length that is $5\frac{1}{2}$ inches broad will make a square foot? *Ans.* $26\frac{3}{11}$ *inches.*

16. A merchant bought 10 packages of cloth, at 4 guineas for 8 yards; each package contained 9 parcels, each parcel, 10 pieces, and each piece, 40 yards; what did the whole cost the merchant, and what per yard?

Ans. L. 25200 at 14*s. a yard.*

17. A borrowed of B *L.* 1400 for 5 months, promising to do him the like kindness; sometime after, B had occasion for *L.* 500; how long may he keep it to balance the favour? *Ans.* 14 *months.*

18. A wall, which was to be built to the height of 18 feet, was raised 6 feet by 8 men in 6 days; how many men must be employed to finish the wall in 4 days, at the same rate of working? *Ans.* 24 *men.*

19. If $\frac{7}{8}$ of a yard of cloth cost *L.* $\frac{4}{5}$; what will $20\frac{1}{2}$ yards cost? *Ans. L.* 18 14*s.* $10\frac{3}{7}$*d.*

20. If $\frac{4}{5}$ of a yard of cambric cost a dollar; what will $\frac{6}{7}$ of an English ell cost? *Ans.* $1\frac{19}{35}$ *dollar.*

21. The Earth, being 360 degrees in circumference, turns round on its axes once in 24 hours; how far does it turn in one minute in the 47th degree of latitude, the degree of longitude, in this latitude, being $47\frac{59}{120}$ statute miles? *Ans.* $11\frac{127}{433}\frac{71}{40}$ *statute miles.*

22. If a staff, $4\frac{1}{2}$ feet long, cast a shade on level ground, $8\frac{3}{4}$ feet; what is the height of that steeple, whose shade, at the same time, measures $225\frac{2}{3}$ feet?

Ans. $116\frac{2}{3}\frac{7}{7}$ *feet.*

23. Suppose a tax of 2000 dollars be laid on a town, and the inventory of all the estates in the town amounts to 45000 dollars; what must A pay, whose estate is 465 dollars? *Ans.* 20 *D.* 66 *c.* $6\frac{2}{3}$ *m.*

24. There is a cistern, having a pipe, which will empty it in 25 hours; how many pipes of the same capacity, will empty it in 25 minutes?

Ans. 60.

25. A person, having $\frac{1}{7}$ of $\frac{3}{8}$ of a Bank, sells $\frac{4}{5}$ of his share for 1000 dollars; what is the value of said Bank?

Ans. 120000 *dollars.*

26. There was a certain building completed in 18 months by 80 workmen, but the same being demolished, it is required to be rebuilt in 6 weeks; I demand how many men must be employed about it?

Ans. 960 *men.*

27. There is a cistern, having 4 cocks; the first will empty it in 15 minutes, the second in 20, the third in 30, and the fourth in 40 minutes; in what time will all four, running together, empty it?

Ans. $5\frac{1}{2}\frac{4}{1}$ *min.*

28. If a suit of clothes can be made of $4\frac{1}{2}$ yards of cloth, $1\frac{1}{4}$ yard wide; how many yards of cassimere, $\frac{7}{8}$ yard wide, will it require for the same person?

Ans. 9 *yards.*

29. A and B depart from the same place, and travel the same road; but A goes 8 days before B, at the rate of 20 miles a day; B follows at the rate of 24 miles; in what time and distance will he overtake A?

Ans. B must travel 40 *days and* 960 *miles.*

30. Bought 4 tuns of oil for 600 dollars; 128 gallons of which leaked out: how must I sell the remainder per gallon so as to lose 50 dollars by the oil?

Ans. 62 *c.* 5 *m.*

31. If $\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{1}{4}$ of $\frac{4}{5}$ of a ship cost $\frac{1}{4}$ of $\frac{1}{7}$ of $\frac{5}{7}$ of

$\frac{5}{6}$ of her cargo, valued at 600000 dollars; what did the whole ship cost? *Ans.* 125000 *dollars.*

32. If 60 men can perform a piece of work in 20 days, how many men can accomplish another piece of work, five times as large in a fourth part of the time?

Ans. 1200 *men.*

33. A merchant bought a number of Bales of velvet, each containing $166\frac{2}{3}$ yards, at the rate of 5 dollars for 4 yards, and sold them at the rate of 7 dollars for 5 yds. and gained 400 dollars by the sale; how many bales were there? *Ans.* 16 *bales.*

34. If $\cdot 6$ *lb.* of tea cost *L.* $\cdot 26$; what will $10 \cdot 90$ *lbs.* cost? *Ans. L.* 4 7*s.* 3*d.* $1\frac{1}{11}$ *qrs.*

35. A can do a piece of work alone in 10 days, and B in 15; set them both about it together; in what time will it be finished? *Ans.* 6 *days.*

36. A cistern, for water, has three cocks to supply it, by the first it may be filled in $\frac{1}{2}$ hour, by the second in $\frac{1}{3}$ hour, and by the third in $\frac{1}{4}$ hour; it has likewise a discharging cock, by which it may, when full, be emptied in $\frac{1}{4}$ hour; now if these four cocks be all left open when the water comes in; in what time will the cistern be filled? *Ans.* 1 *hour* 12 *min.*

37. A can do a piece of work alone in 16 days, and A and B together in 12 days; in what time can B do it by himself? *Ans.* 48 *days.*

38. A Gentleman divided his fortune among his sons, to A he gave 12 dollars as often as to B 7, and to C but 5 dollars as often as to B 9, and yet C's portion was 4550 dollars; what was the whole estate?

Ans. 26780 *dollars.*

39. A, B, and C can complete a piece of work in 24 days; A can do it alone in 48 days, and B in 68; in what time can C do it alone?

Ans. $163\frac{1}{3}$ *days.*

40. A and B are on opposite sides of a circular field 400 rods about; they begin to go round it, both the same way, at the same instant of time; A goes 22 poles in 2 minutes, and B 45 poles in 4 minutes; how many times will they go round the field, before the swifter overtakes the slower? *Ans.* $22\frac{1}{2}$ *times.*

COMPOUND PROPORTION.

Compound Proportion or *the Double Rule of Three* is that, which teaches to resolve such questions, as require two or more statings in *Simple Proportion.*

Rule for Stating.

Write down the number* which is of the same kind with the answer, for the third term; then take one of the numbers or terms of supposition and one of the demanding terms of the same kind, placing one of them for a first term and the other for a second, according to the directions given in simple proportion. Do the same with another term of supposition and its correspondent demanding term, and so on; writing the terms, which belong to the first and second places, under one another.

First Method of Operation.

Take the upper terms, in the same order in which they stand, for the first stating of the rule of three; then take the fourth number resulting from the first stating, together with the next two terms in the general stating, in the same order as they stand, for the second stating, and so

* That is, the term of supposition.

on; always making the fourth number, resulting from each simple stating, the third term of the next. So the last resulting number will be the answer required.

Second Method.

Multiply together all the terms in the first place, and also all the terms in the second place: then multiply the latter product by the third term, and divide the result by the former product, and the quotient will be the answer.

NOTE 1. It is generally best to work by the latter method. And after the stating, and before the commencement of the operation, if one of the first terms, and either one of the second terms, or the third term, can be measured by any number, let them be divided, and the quotients used instead of them.

NOTE 2. The first and second terms of each line, if compounded, must be reduced to the lowest, and to the same denomination.

EXAMPLES.

1. If L.100 gain L. 6 in a year; what will L. 800 gain in 3 months?

General Stating.

mo.	*mo.*	*L.*
12	3	6
100	800	

$12 \div 6 = 2$, $6 \div 6 = 1$; $100 \div 100 = 1$, and $800 \div 100 = 8$. ∴

First Method.

mo.	*mo.*	*L.*	*L.*	*L.*	*L.*	*s.*
2	3	1	1	8	1	10
	1					20
	2)3					30
	L. 1	10				8
					2,0)24,0	

L. 12 the Ans.

Second Method.

mo.	*mo.*	
2	3	*L.*
1	8	: : 1
2	24	: : 1

1

2)24

L. 12 *the Ans. as before.*

2. If 16 horses can eat up 9 bushels of oats in 6 days, how many horses can eat 168 bushels in 7 days at the same rate? Ans. 256 *horses.*

3. If a man travel 120 miles in 6 days, when the days are 12 hours long; how many days, of 16 hours long, will he require to travel 2880 miles?

Ans. 108 *days.*

4. If 8 men spend *L.* 64 in 13 weeks; what will 48 men spend in 52 weeks? Ans. *L.* 1536.

5. A man put out *L.* 450 at interest, and when it had continued 9 months, he received for principal and interest L. 473 12 6: I demand at what rate *per Cent, per Annum* he received interest? Ans. *L.* 7 *per Cent.*

6. What is the interest of 2400 dollars for 252 days, at 6 *per Cent, per Annum?* Ans. 99 *d.* 41 *c.* $9\frac{1}{7}\frac{3}{3}$ *m.*

7. There was a certain Edifice completed in 2 years by 10 workmen; and it is requested that 2 others, just like it, should be built in $7\frac{1}{2}$ months; I demand the number of men which must be employed about them.

Ans. 64 *men.*

8. If 20 men can perform a piece of work in 40 days, how many men can do another piece of work, eight times as large in a fifth part of the time? Ans. 800.

9. If the freight of $10\frac{1}{2}$ tons of iron 13 leagues, cost L. $13\frac{1}{2}$; what must be paid for the freight of $155\frac{1}{4}$ tons 80 leagues? Ans. L. 1241 8 $8\frac{8}{11}$d.

10. If 248 men, in 5 days of 11 hours each, can dig a trench 230 yards long, 3 wide, and 2 deep; in how many days of 9 hours long, will 24 men dig a trench of 840 yards long, 5 wide, and 3 deep?

Ans. $576\frac{118}{567}$ *days.*

CONJOINED PROPORTION.

Conjoined Proportion is the comparing together of several coins, weights, or measures of different countries in the same question; or it is the joining together of several ratios, and the finding, from the ratios of all the antecedents and their respective consequents, the ratio of the first antecedent and the last consequent by *illation.*

CASE 1.

When it is required to find how many of the last kind of coin, weight, or measure, expressed in the question, are equal to a given number of the first.

RULE.

Multiply into one another all the antecedents for the first term, and all the consequents for the second, and make the given number the third; then find the fourth term, or proportional, which will be the answer.

NOTE 1. The solutions of questions, in this and the next Case, may often be much shortened by expunging equal numbers in the columns, or in the first column and

third term, and by abbreviating those that are commensurable.

NOTE 2. The method of proof is by so many statements in the rule of three as the nature of the question requires.

EXAMPLES.

1. If 30lbs. at Boston make 27lbs. at Amsterdam; 270lbs. at Amsterdam, 336lbs. at Thoulouse; how many pounds at Thoulouse are equal to 150lbs. at Boston?

Ant.	*Con.*		
30	27		
270	336		
8100	9072	::	150
	150		
	45360		
	9072		
	81,00)13608,00(168lbs. the Ans.		
	81		
	550		
	486		
	648		
	648		
	...		

Or by abbreviation.

30	:	27	::	150	3	:	27	::	15	1	: 9 :: 15
270	:	336			270	:	336			270	: 336
1	:	9	::	15	1	:	9	::	5	1	: 9 :: 1
90	:	112			30	:	112			6	:112
1	:	9	::	1							
3	:	56			1	:	3	::	56	:	168 *as before.*

2. If 10 braces at Leghorn be equal to 5 vares at Lisbon; 10 vares at Lisbon equal to 20 braces at Lucca; how many braces at Lucca are equal to 200 braces at Leghorn? Ans. 200 braces at Lucca.

CASE 2.

When it is required to find how many of the first kind of coin, &c. mentioned in the question, are equal to a given number of the last.

Rule.

Make the rectangle of the consequents the first term, that of the antecedents the second, and the given number the third; and the fourth, resulting, will be the answer.

Examples.

1. If 6lbs. at Portsmouth make 5lbs. at Amsterdam, 50lbs. at Amsterdam, 60lbs. at Paris; how many pounds at Portsmouth are equal to 160lbs. at Paris?

Con. *Ant.*
5 : 6
60 : 50

300 : 300 : : 160
300

3,00)480,00

160 lbs. the Ans.

Or by abbreviation.

5 : 6
60 : 50 : : 160
10 : 10 : : 160

1 : 6
60 : 10 : : 160
1 : 10 : : 16 : 160 Ans.

2. If 20lbs. in America be equal to 19lbs. Flemish, and 95lbs. Flemish equal to 125lbs. at Bolognia; how many pounds in America are equal to 250lbs. at Bolognia? Ans. 200 pounds.

PRACTICE.

Practice is an abbreviation of the rule of three, when the first term is an unit; and has its name from its constant use among merchants and tradesmen, being an easy and concise method of answering most questions that generally occur in trade and business.

An *aliquot* part of any number is such a part of it, as, being taken a certain number of times, will exactly make that number.

TABLES.

d.		*s.*
6	=	$\frac{1}{2}$
4	=	$\frac{1}{3}$
3	=	$\frac{1}{4}$
2	=	$\frac{1}{6}$
$1\frac{1}{2}$	=	$\frac{1}{8}$
1	=	$\frac{1}{12}$

s.	*d.*		*L.*
10		=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$
5		=	$\frac{1}{4}$
4		=	$\frac{1}{5}$
3	4	=	$\frac{1}{6}$
2	6	=	$\frac{1}{8}$
2		=	$\frac{1}{10}$
1	8	=	$\frac{1}{12}$

lb.		*Cwt.*
56	=	$\frac{1}{2}$
28	=	$\frac{1}{4}$
16	=	$\frac{1}{7}$
14	=	$\frac{1}{8}$
8	=	$\frac{1}{14}$
7	=	$\frac{1}{16}$

General Rule.

Suppose the price of the given quantity to be L. 1, 1s. &c. as is most convenient; and the quantity itself will be the answer at the supposed price. Then divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients, belonging to each, will be the answer required.

Note 1. When there is a fraction, or an inferior denomination in the given quantity, take the same part of the price, that the fraction, or inferior denomination, is of an unit of the said quantity, and add it to the price of the whole number.

Note 2. The method of proof is by the rule of three, compound multiplication, or by inverting the question.

EXAMPLE.

What is the value of 365 yards of chintz at 3s. 10½d. per yard? 365 *Ans. at L.*1.

3s. 4*d.* *is* ⅓	=	60	16	8	*do. at* 3s. 4d.
4*d.* *is* $\frac{1}{15}$	=	6	1	8	*do. at* 4d.
2*d.* *is* ½	=	3	0	10	*do. at* 2d.
½*d.* *is* ¼	=	0	15	2½	*do. at* 0½d.
	L.	70	14	4½	*do. at* 3 10½ *the full price.*

Ans. L. 70 14 4½.

N. B. A few of the many cases, which may occur, will, with the particular rule belonging to each, be sufficient to illustrate the general Rule.

CASE 1.

When the price is less than a penny.

RULE.

Divide the given number by the aliquot part of a penny, and the quotient will be the answer in pence, which bring into pounds.

EXAMPLES.

¼*d.*	¼	4565 at ¼*d.*
	12	1141¼
	2,0	9,5 1
		*L.*4 15 1¼ *the Ans.*

2. 6843 at ¼*d.* ..... Ans. *L.* 7 2*s.* 6¾*d.*
3. 8000 at ½*d.* ..... Ans. *L.* 16 13*s.* 4*d.*
4. 8765 at ¾*d.* ..... Ans. *L.* 27 7*s.* 9¼*d.*

CASE 2.

When the price is an aliquot part of a shilling.

RULE.

Divide the given quantity by the aliquot part, and the quotient is the answer in shillings, which reduce to pounds.

EXAMPLES.

1*d.*	$\frac{1}{12}$	9876 at 1*d.*
	2,0	82,3

*L.*41 3*s. the Ans.*

2. 8765 at $1\frac{1}{2}d.$ ..... Ans. *L.* 54 15*s.* $7\frac{1}{2}d.$
3. 7654 at 2*d.* ..... Ans. *L.* 63 15*s.* 8*d.*
4. 6543 at 3*d.* ..... Ans. *L.* 81 15*s.* 9*d.*
5. 5432 at 4*d.* ..... Ans. *L.* 90 10*s.* 8*d.*
6. 4321 at 6*d.* ..... Ans. *L.* 108 0*s.* 6*d.*

CASE 3.

When the price is a part, but not an aliquot part, of a shilling.

RULE.

Divide the given quantity by some aliquot part of a shilling; then divide by the part of a shilling, or the part of the said aliquot part the rest is, and the sum of the quotients will be the answer in shillings, which reduce into pounds as before.

EXAMPLES.

2*d.*	$\frac{1}{6}$	3210 at $2\frac{1}{4}d.$
$\frac{1}{2}$	$\frac{1}{4}$	535
$\frac{1}{4}$	$\frac{1}{2}$	133 9
		66 $10\frac{1}{2}$
2,0		73,5 $7\frac{1}{2}$

L. 36 15*s.* $7\frac{1}{2}d.$ *Ans.*

2. 2101 at $3\frac{1}{4}d.$ ..... Ans. L. 28 9*s.* $0\frac{1}{4}d.$
3. 1012 at $3\frac{1}{2}d.$ ..... Ans. L. 14 15*s.* 2*d.*
4. 2000 at $3\frac{3}{4}d.$ ..... Ans. L. 31 5*s.*
5. 2123 at $4\frac{1}{4}d.$ ..... Ans. L. 37 11*s.* $10\frac{1}{4}d.$
6. 2234 at $4\frac{1}{2}d.$ ..... Ans. L. 41 17*s.* 9*d.*
7. 2345 at $4\frac{3}{4}d.$ ..... Ans. L. 46 8*s.* $2\frac{1}{4}d.$
8. 3456 at 5*d.* ..... Ans. L. 72.
9. 4567 at $5\frac{1}{4}d.$ ..... Ans. L. 99 18*s.* $0\frac{1}{4}d.$
10. 5678 at $5\frac{1}{2}d.$ ..... Ans. L. 130 2*s.* $5\frac{1}{2}d.$
11. 6789 at $5\frac{3}{4}d.$ ..... Ans. L. 162 13*s.* $0\frac{1}{4}d.$
12. 7890 at $6\frac{1}{4}d.$ ..... Ans. L. 205 9*s.* $4\frac{1}{2}d.$
13. 8900 at $6\frac{1}{2}d.$ ..... Ans. L. 211 0*s.* 10*d.*
14. 9000 at $6\frac{3}{4}d.$ ..... Ans. L. 253 2*s.* 6*d.*
15. 9001 at 7*d.* ..... Ans. L. 262 10*s.* 7*d.*
16. 9012 at $7\frac{1}{4}d.$ ..... Ans. L. 272 4*s.* 9*d.*
17. 9123 at $7\frac{1}{2}d.$ ..... Ans. L. 285 1*s.* $10\frac{1}{2}d.$
18. 9234 at $7\frac{3}{4}d.$ ..... Ans. L. 298 3*s.* $7\frac{1}{2}d.$
19. 9345 at 8*d.* ..... Ans. L. 311 10*s.*
20. 9456 at $8\frac{1}{4}d.$ ..... Ans. L. 325 1*s.*
21. 9567 at $8\frac{1}{2}d.$ ..... Ans. L. 338 16. $7\frac{1}{2}d.$
22. 9678 at $8\frac{3}{4}d.$ ..... Ans. L. 352 16*s.* $10\frac{1}{2}d.$
23. 9789 at 9*d.* ..... Ans. L. 367 1*s.* 9*d.*
24. 9890 at $9\frac{1}{4}d.$ ..... Ans. L. 381 3*s.* $6\frac{1}{2}d.$
25. 9900 at $9\frac{1}{2}d.$ ..... Ans. L. 391 17*s.* 6*d.*
26. 8000 at $9\frac{3}{4}d.$ ..... Ans. L. 325.
27. 7000 at 10*d.* ..... Ans. L. 291 13*s.* 4*d.*
28. 6000 at $10\frac{1}{4}d.$ ..... Ans. L. 256 5*s.*
29. 5000 at $10\frac{1}{2}d.$ ..... Ans. L. 218 15*s.*
30. 4000 at $10\frac{3}{4}d.$ ..... Ans. L. 179 3*s.* 4*d.*

31. 3000 at 11*d.* ..... Ans. L. 137 10*s.*
32. 2000 at 11¼*d.* ..... Ans. L. 93 15*s.*
33. 2008 at 11½*d.* ..... Ans. L. 96 4*s.* 4*d.*
34. 2050 at 11¾*d.* ..... Ans. L. 100 7*s.* 3½*d.*

CASE 4.

When the price is more than one shilling, but less than two.

Rule.

Let the part or parts be taken only with so much of the given price as is more than one shilling, and let the given quantity stand for shillings, which must be added with the rest.

Examples.

1*d.*	½	4321 at 13*d.*
		360 1
2,0		468,1 1

L. 234 1*s.* 1*d.* *the Ans.*

2. 4123 at 13¼*d.* ..... Ans. L. 227 12*s.* 5¼*d.*
3. 4080 at 13½*d.* ..... Ans. L. 229 10*s.*
4. 4700 at 13¾*d.* ..... Ans. L. 269 5*s.* 5*d.*
5. 8090 at 14*d.* ..... Ans. L. 471 18*s.* 4*d.*
6. 8880 at 15¼*d.* ..... Ans. L. 564 5*s.*
7. 9990 at 16½*d.* ..... Ans. L. 686 16*s.* 3*d.*
8. 5600 at 17*d.* ..... Ans. L. 396 13*s.* 4*d.*
9. 6060 at 17¼*d.* ..... Ans. L. 422 18*s.* 9*d.*
10. 7007 at 18¼*d.* ..... Ans. L. 532 16*s.* 5¼*d.*
11. 3965 at 19*d.* ..... Ans. L. 313 17*s.* 11*d.*
12. 2345 at 20¾*d.* ..... Ans. L. 202 14*s.* 10¾*d.*
13. 1234 at 21½*d.* ..... Ans. L. 110 10*s.* 11*d.*
14. 4321 at 22¼*d.* ..... Ans. L. 400 11*s.* 10¼*d.*
15. 12345 at 23¾*d.* ..... Ans. L. 1221 12*s.* 9¾*d.*

CASE 5.

When the price is any number of shillings under 20.

RULE.

1. *When the price is an even number*, multiply the given quantity by $\frac{1}{2}$ of it, doubling the unit figure of the product for shillings, and the rest are pounds.

2. *When the price is an odd number*, find for the greatest even number as before, to which add $\frac{1}{10}$ of the given quantity for the odd shillings, and the sum is the answer.

EXAMPLES.

1. 6964 at 4*s*.

2

L. 1392 16*s*. *Ans*.

2. 9876 at 5*s*.

2

1975 4

1*s*. is $\frac{1}{10}$ = 493 16

L. 2469 *Ans*.

3. 1000 at 6*s*. ..... Ans. L. 300.
4. 2040 at 7*s*. ..... Ans. L. 714.
5. 8765$\frac{1}{4}$ at 8*s*. ..... Ans. L. 3506 2*s*.
6. 6412 at 9*s*. ..... Ans. L. 2885 8*s*.
7. 4005 at 10*s*. ..... Ans. L. 2002 10*s*.
8. 909 at 11*s*. ..... Ans. L. 499 19*s*.
9. 876$\frac{1}{2}$ at 12*s*. ..... Ans. L. 525 18*s*.
10. 765 at 13*s*. ..... Ans. L. 497 5*s*.
11. 654 at 14*s*. ..... Ans. L. 457 16*s*.
12. 543 at 15*s*. ..... Ans. L. 407 5*s*.
13. 432 at 16*s*. ..... Ans. L. 345 12*s*.
14. 3210 at 17*s*. ..... Ans. L. 2728 10*s*.
15. 2100 at 18*s*. ..... Ans. L. 1890.
16. 1234 at 19*s*. ..... Ans. L. 1172 6*s*.

CASE 6.

When the price is shillings and pence, which make some aliquot part of a pound.

RULE.

Divide the given quantity by the aliquot part, and the quotient is the answer in pounds.

EXAMPLES.

s.	*d.*		
1	8	$\frac{1}{12}$	7612 at 1*s.* 8*d.*
			L.634 6*s.* 8*d.* *the Ans.*

2. 5063 at 2*s.* 6*d.* ..... Ans. L. 632 17*s.* 6*d.*
3. 4000 at 3*s.* 4*d.* ..... Ans. L. 666 15*s.*
4. 2345 at 6*s.* 8*d.* ..... Ans. L. 781 13*s.* 4*d.*

CASE 7.

When the price is shillings and pence, which make no aliquot part of a pound.

RULE.

Multiply the given quantity by the shillings, then, take parts with the rest; and the sum will be the answer in shillings.

EXAMPLES.

1d.	$\frac{1}{12}$	960 at 4s. 1d.
		4
		3840
		80
	2,0	392,0

L. 196 *the Ans.*

2. 942 at 5*s*. 2*d*. ..... Ans. L. 243 7*s*.
3. 812 at 6*s*. 3*d*. ..... Ans. L. 253 15*s*.
4. 800 at 7*s*. 4¼*d*. ..... Ans. L. 294 3*s*. 4*d*.
5. 785 at 8*s*. 5*d*. ..... Ans. L. 330 7*s*. 1*d*.
6. 567 at 9*s*. 6*d*. ..... Ans. L. 269 6*s*. 6*d*.
7. 653¼ at 10*s*. 7½*d*. ..... Ans. L. 347 6*s*. 1*d*.
8. 356 at 11*s*. 8*d*. ..... Ans. L. 207 13*s*. 4*d*.
9. 584 at 12*s*. 9¼*d*. ..... Ans. L. 374 2*s*. 6*d*.
10. 432⅞ at 13*s*. 10*d*. ..... Ans. L. 299 8*s*. 1¼*d*.
11. 312 at 14*s*. 11¼*d*. ..... Ans. L. 233 0*s*. 6*d*.

CASE 8.

When the price is pounds and Shillings.

Rule.

Multiply the given quantity by the pounds and proceed with the shillings according to Case 5th. and the sum is the answer in pounds.

Examples.

1. 54 at L. 4. 8*s*.

4

216
21 12

L. 237 12 *the Ans.*

2. 47 at L. 5. 9*s*.

5

235
18 16
2 7

L. 256 3 Ans.

3. 38 at L. 6 10*s*. ..... Ans. L. 247.
4. 60 at L. 7 11*s*. ..... Ans. L. 453.

5. 100 at L. 8 12*s*. ..... Ans. L. 860.
6. 23 at L. 9 13*s*. ..... Ans. L. 221 19*s*.
7. 30 at L. 10 14*s*. ..... Ans. L. 321.
8. 409 at L. 21 15*s*. ..... Ans. L. 8895 15*s*.

CASE 9.

When the price is pounds, shillings, and pence.

RULE.

1. *If the shillings and pence be the aliquot part of a pound;* multiply by the pounds; then divide by the aliquot part; and their sum is the answer in pounds.

2. *If the shillings and pence be not the aliquot part of a pound;* reduce the pounds into shillings; multiply the quantity by the shillings; then take parts with the rest; and the sum will be the answer in shillings.

EXAMPLES.

	d.		
1s8	$\frac{1}{12}$	16 at L. 2. 1*s*. 8*d*.	
		2	
		32	
		1 6 8	

L. 33 6 8 *the Ans.*

2*d*.	$\frac{1}{6}$	23 at L. 3 2*s*.	$2\frac{1}{4}d$.
		62	20
		46	62
		138	
		1426	
$\frac{1}{4}$	$\frac{1}{8}$	3 10	
		0 $5\frac{1}{4}$	
	2,0	143,0 $3\frac{1}{4}$	

L. 71 10 $3\frac{1}{4}$ *the Ans.*

3. 24 at L. 3 2*s*. 6*d*. . . . . Ans. L. 75.
4. 25 at L. 4 3*s*. 4*d*. . . . Ans. L. 104 3*s*. 4*d*.
5. 26 at L. 4 3*s*. 9*d*. . . . Ans. L. 108 17*s*. 6*d*.
6. 27¼ at L. 5 6*s*. 8*d*. . . . Ans. L. 145 6*s*. 8*d*.
7. 28 at L. 6 7*s*. 6*d*. . . . . Ans. L. 178 10*s*.
8. 125½ at L. 1 10*s*. 5*d*. . . . Ans. L. 190 17*s*. 3½*d*.
9. 234 at L. 2 11*s*. 7*d*. . . . Ans. L. 603 10*s*. 6*d*.
10. 345¼ at L. 7 15*s*. 11¼*d*. . . Ans. L. 2696 9*s*. 9¼*d*.

TARE AND TRETT.

Tare and *Trett* are rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance to the buyer for the weight of the box, barrel, bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per Cwt. or at so much in the gross weight.

Trett is an allowance of 4lb. in every 104lb. for waste, dust, &c.

Cloff is an allowance of 2lb. upon every 3 Cwt.

Gross weight is the whole weight of the goods, together with the box, barrel, bag, &c. that contains them.

Suttle is the weight which remains, after part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made

CASE 1.

When the tare is at so much per box, barrel, bag, &c.

RULE.

Multiply the number of boxes, barrels, &c. by the

tare, and subtract the product from the gross, and the remainder is the neat weight required.

EXAMPLES.

1. In 8 frails of raisins, each weighing 5Cwt. 3qr. 12lb. tare 22lb. per frail, how much neat ?

$22 \times 8 = 1$ *Cwt.* 2 *qr.* 8 *lb.*

Cwt.	*qr.*	*lb.*	
5	3	12	
		8	
46	3	12	*gross.*
1	2	8	*tare.*
45	1	4	*the Ans.*

2. In 200 barrels of figs, each 1Cwt. 1qr. 16lb. gross, tare 12lb. *per barrel*, how many pounds neat ?

Ans. 28800lb.

3. What is the neat weight of 17 bags of cotton, each weighing 5Cwt. 3qr. 23lb. gross, tare 23lb. *per bag ?*

Ans. 97*Cwt.* 3*qr.*

4. What is the neat weight of 13hhds. of tobacco, each 6Cwt. 1qr. 15lb. gross. tare 100lb. *per hhds ?*

Ans. 71*Cwt.* 1*qr.* 15*lb.*

CASE 2.

When the tare is a certain weight per cwt.

RULE.

Divide the gross weight by the aliquot parts of a cwt. subtract the sum of the quotients from the gross, and the remainder will be the neat weight.

EXAMPLES.

1. Gross 124 cwt. 1qr. 20lb. tare 16lb. *per cwt.* how much neat ?

lb.		*Cwt. qr. lb.*	
14	$\frac{1}{7}$	124 1 20	*gross.*
2	$\frac{1}{7}$	15 2 6	
		2 0 $24\frac{6}{7}$	
		17 3 $2\frac{6}{7}$	*tare.*
		106 2 $17\frac{1}{7}$	*the Ans.*

2. What is the neat weight of 12 barrels of pot-ash, each weighing 221lb. gross, tare 14lb. *per cwt ?*

Ans. $2320\frac{1}{2}$*lb.*

3. What is the neat weight of 194Cwt. 2qr. 14lb. gross, tare 20lb. *per Cwt.?*

Ans. 159*Cwt.* 3*qr.* $13\frac{1}{4}$*lb.*

4. In 17hhds. of tobacco, each weighing 6cwt. 2qr. 13lb. gross, tare 21lb. *per cwt.* how much neat ?

Ans. 91*Cwt.* 1*qr.* $15\frac{1}{16}$*lb.*

CASE 3.

When trett is allowed with tare.

RULE.

Divide the suttle weight by 26, and the quotient is the trett, which subtract from the suttle, and the remainder is the neat.

EXAMPLES.

1. In 8cwt. 2qr. gross, tare 16lb. *per cwt.* and trett 4lb. *per* 104lb. what neat weight ?

lb.		*Cwt. qr.*		
16	$\frac{1}{7}$	8	2	*gross.*
		1	0 24	*tare.*
4	$\frac{1}{26}$	7	1 4	*suttle.*
		–	1 $3\frac{5}{13}$	*trett.*
		7	0 $0\frac{8}{13}$	*the Ans.*

2. In 25cwt. 2qr. gross, tare 8lb. *per cwt.* and trett as usual, how much neat ?

Ans. 22*Cwt.* 3*qr.* 2*lb.*

3. In 14 casks of prunes, each weighing 3cwt. 1qr. 5lb. gross, tare 17½lb. *per cwt.* and trett as ausul, how much neat ? Ans. 37*cwt.* 1*qr.* 19*lb.* 2*oz.* $10\frac{6}{11}$*dr.*

4. What is the neat weight of 4hhds. of sugar, weighing as follow : the 1st. 3cwt. 2qr. 27lb. gross, tare 40lb. ; the 2d. 3cwt. 3qr. gross, tare 47lb. ; the 3d. 4cwt. 3qr. 25lb. gross, tare 52lb. ; and the 4th. 5cwt. 3qr. 19lb. gross, tare 65lb, and allowing trett as usual ?

Ans. 15*Cwt.* 3*qr.* $19\frac{17}{28}$*lb.*

CASE 4.

When tare, trett, and cloff are allowed.

Rule.

Deduct the tare and trett by the preceding Cases ; divide the suttle by 168, and the quotient is the cloff, which subtract from the suttle, and the remainder is the neat.

Examples.

1. In 44cwt. 2qr. 8lb. gross, tare 14lb. *per cwt.* trett and cloff as usual ; how much neat ?

lb.		*Cwt.*	*qr.*	*lb.*	
14	$\frac{1}{8}$	44	2	8	*gross.*
		5	2	8	*tare.*
4	$\frac{1}{28}$	39	0	0	
		1	2	0	*trett.*
2	$\frac{1}{168}$	37	2	0	*suttle.*
		—	—	25	*Cloff.*
		37	1	3	*the Ans.*

2. In 39cwt. gross, tare 8lb. *per cwt.* trett and cloff as usual ; what is the neat weight?

Ans. 34*Cwt.* 2*qr.* $12\frac{11}{14}$*lb.*

3. What is the neat weight of 78 cwt. gross, tare 7lb. *per cwt.* trett and cloff as usual ?

Ans. 69*Cwt.* 3*qr.* 16*lb.* 2*oz.*

4. What is the neat weight of a hhd. of sugar, weighing 8cwt. 1qr. 10lb. gross, tare 14lb. *per cwt.* trett and cloff as usual ?

Ans. $6Cwt.\ 3qr.\ 20\frac{19}{175}lb.$

BARTER.*

Barter is the exchanging of one commodity for another ; and directs merchants and traders so to proportion their goods, that neither party may sustain loss.

EXAMPLES.

1. How much sugar, at 8*d. per lb.* must be given in barter for 40*Cwt.* of tobacco, at L. 3 *per Cwt.* ?

Cwt. Cwt. L.
1 : 40 :: 3 :
3

L. 120

d. L. lb.
8 : 120 :: 1 :
20

2400
12

8)28800

7)3600

4)514

4)128 16

Cwt. 32 0 16*lb. the Ans.*

* Barter is evidently, only, an application of *simple proportion.*

2. A delivered to B $1\frac{1}{2}$ tun of rum at 6*s.* 8*d. per gallon*, for 252 yards of cloth; what was the cloth *per yard?* Ans. 10 *shillings.*

3. How many reams of paper, at $3\frac{1}{2}$ dollars *a ream*, must be given in barter for 50 yards of cloth, at $5\frac{1}{4}$ dollars *per yard?* Ans. 75 *reams.*

4. A barters, with B, 160 bushels of barley, at 50 cents *per bushel*, for 200 bushels of oats, at 36 cents *per bushel;* which must receive money and how much? Ans. B *must pay* A 8 *dollars.*

5. A and B barter; A has 3 *hhds.* of rum at 8*s. per gallon*, for which B gives him L. 40 in cash, and the rest in boards at L. 2 10*s. a thousand;* how many feet of boards must A receive? Ans. 14240 *feet.*

6. A has linen worth 4*s.* 6*d. a yard* ready money, but in barter he will have 5*s.* B has satin worth 12*s.* 6*d. a yard* ready money, at what price should B's satin be rated in barter? Ans. 13*s.* $10\frac{2}{7}$*d.*

7. A has 400 yards of linen, at 2*s.* 6*d.* per yard ready money, which he barters with B, at 3*s.* per yard, taking India cotton, at 2*s. per yard* which is worth but 1*s.* 10*d.*: how many yards of India cotton will pay for the linen; who has the best bargain, and how much, both in the whole and *per cent?*

Ans. 600 *yards of India cotton; A has the best bargain; A's whole gain is L.* 5, *and the gain per cent, L.* 10.

8. A has cambric at 6*s. a yard*, but in barter he must have 7*s.* 6*d.* B has 4000 yards of tow cloth at 1*s. a yard* for the money, and would have of A L. 60 in cash, and the rest in cambric; what rate does the tow cloth bear in barter *per yard*, and what quantity of cambric must A give B?

Ans. *The rate of the tow cloth is* 1*s.* 3*d. a yard, and B must receive* $466\frac{2}{3}$ *yards of cambric with L.* 60 *in cash.*

LOSS AND GAIN.*

Loss and Gain is a rule by which merchants and t
ers discover their profit or loss in the buying or se
of goods, and teaches them to raise or fall the pric
their commodities, so as to gain or lose so much
cent, &c.

EXAMPLES.

1. If I buy flannel at 1*s.* 6*d.* *per yard* and sell
gain at 1*s.* 9*d.*: what do I gain *per cent?*

s. *d.*	*s.* *d.*		*L.*		*d.*
1 9					
1 6	1 6	:	100	::	3 :
—	12		20		
3	—		—		
	18		2000		
			12		
			—		
			24000		
			3		
			—		12
			18)72000(		4000
			72		—
			—		2,0)33,3 4
			..		—

L. 16 13 4*d.* *the*

2. At 2*s.* 6*d.* in the pound profit; how much *per c*
Ans. L. 12

3. If I buy a hhd. of molasses for L. 9 9*s.* and s

* Questions in *loss* and *gain* are performed by *simple proportion*

again, at 4*s.* a gallon; do I gain or lose, and what *per cent.?*

Ans. *whole gain* L. 3 3*s.* *and* L. 33 6*s.* 8*d.* *gain per cent.*

4. Bought coffee at 1*s.* 9*d.* *per lb.* and sold it, at L.11 4*s.* *per cent.*; what was the gain or loss *per cent?*

Ans. *gain* L. 14 5*s.* $8\frac{4}{7}$*d.*

5. B merchant bought 40 yards of broad cloth, at 30*s.* a yard, and 30 yards of *do.* at 20*s.* *per yard*, and sold them upon an everage at 25*s.* a yard; did he gain or lose, and what *per cent.?*

Ans. *he lost L.* 2 10*s.* *in the gross, and L.* 2 15*s.* $6\frac{4}{7}$*d.* *per cent.*

6. If 1 *cwt.* weight of cotton cost L. 14.; how must it be sold *per lb.* to lose L. 5 *per cent.?*

Ans. 2*s.* $4\frac{1}{2}$*d.*

7. Bought a pipe of brandy, at 1 dollar per gallon, but, by accident, 26 gallons leaked out; at what rate per gallon must I sell the remainder, to gain upon the whole prime cost, at the rate of 10 *per cent.?*

Ans. 1 *D.* 38 *c.* 6 *m.*

8. If a man buy cloth at 14*s.* a yard, on 8 months credit, and sell it again, at 13*s.* 6*d.* ready money; does he gain or lose, and what *per cent.?*

Ans. *he gains* $\frac{6}{13}$*d.* *on a yard, and* 5*s.* $8\frac{4}{7}$*d.* *per cent.*

9. If 1lb. of tea be sold at 6*s.* 9*d.* and there is gained L. 12 10*s.* *per cent.*; what did the lb. cost.

Ans. 6*s.*

10. If 15cwt. of flour be sold at 14 dollars *per cwt.* and there is gained 25 *per cent.*; what is the prime cost *per cwt.?* Ans. 11 *D.* 20 *cents.*

11. If wine, sold at 9*s.* per gallon, be L. 15 *per cent.* profit; what would the gain or loss have been *per cent.* if it had been sold at 7*s.* 6*d.* per gallon?

Ans. L. 4 3*s.* 4*d.* *loss per cent.*

12. If I sell brocade at 8*s.* per yard, and thereby lose

L. 30 *per cent.*; what shall I gain or lose *per cent.* if I sell 8 yards of the same for L. 4 7*s.*

Ans. *I shall lose* L. 4 16*s.* $10\frac{1}{2}$*d. per cent.*

13. Bought a number of casks of raisins, at 11 *D.* 20 *cents* per cask, which not proving so good as I expected, I am willing to lose $14\frac{1}{2}$ *per cent.*; how must I sell them per cask? Ans. 9 *D.* 57 *cents.* 6 *mills.*

14. A gentleman bought 20 tuns of molasses for L. 504; the freight and duties came to L. 100, and his own charges to L. 26; how must he sell it per gallon to gain L. 20 *per cent.*? Ans. 3*s.*

15. If I buy silk stockings, at 2 dollars per pair; how long credit must I have to gain 20 *per cent.* when I sell them, at $2\frac{1}{4}$ dollars per pair? Ans. 15 *months.*

16. I sold a horse for 60 dollars, and by so doing, lost 20 *per cent.* whereas I ought in trading to have cleared 30 *per cent.* what was the real value of the horse?

Ans. $97\frac{1}{2}$ *dollars.*

FELLOWSHIP.*

Fellowship is a general rule, by which merchants, &c. trading in company, are so adjusted, that each may have his share of the gain or sustain his share of the loss in proportion to his share of the joint stock, together with the time of its continuance in trade.

* By *Fellowship* a bankrupt's estate may be divided among his creditors; legacies adjusted when there is a deficiency in effects, &c.

THEOREM.

When the times are equal, the shares of the gain or loss are in proportion to the stocks, and when the stocks are equal, the shares of the gain or loss are in proportion to the times; therefore, when neither are equal, the shares are in proportion to their products.

CASE 1.

When either the times or the stocks are equal.

RULE.*

As the whole stock or time is to the whole gain or loss; so is each man's particular stock or time to his particular share of the gain or loss.

Method of Proof.

Add all the shares together, and the sum will be equal to the whole gain or loss, when the work is right.

EXAMPLES.

1. Two persons traded together; A put into stock L. 200, and B L. 250, and they gained L. 300; what is each person's share of the gain?

Whole stock.	*L.*		*L.*	*s.*	*d.*	
As 200+250 :	300 ::	200 :	133	6	8	*A's share.*
		250 :	166	13	4	*B's share.*
		L.	300			*Proof.*

* *When the times are equal;* fay, as the whole ſtock is to the whole gain or loſs; ſo is each man's particular ſtock to his particular ſhare of the gain or loſs. But, *when the ſtocks are equal*; fay, as the whole time is to the whole gain or loſs; ſo is each man's particular time to his particular ſhare of the gain or loſs.

2. A began trade with L. 400, and in 4 months afterwards, he received B as a partner, with a stock of L. 400 more; and when they had traded together 16 months, they found their gain to be *L.* 500; what is the share of each?

		mo.	*L.*	*s.*	*d.*	
Whole time.	*L.*	20	277	15	$6\frac{3}{7}$	*A's share.*
As 20+16	:: 500 ::	16	222	4	$5\frac{1}{7}$	*B's share.*
			L. 500			*Proof.*

3. A and B have gained by trading *L.* 280; A had L. 240 stock in trade 12 months, and B L. 240 in 6 months; what is each person's share of the gain?

Ans. A's share is L. 186 13 4, *and B's L.* 93 6 8*d.*

4. A, B, and C put their money into a joint stock; A put in L. 140, B L. 200, and C L. 280, but by misfortune they lost L. 425 18*s.*; how much must each sustain of the loss?

Ans. A's loss is L. 96 3 5 $0\frac{4}{11}qr$. *B's, L.* 137 7 8 $3\frac{10}{11}qrs$. *and C's, L.* 192 6 10 $0\frac{8}{11}qr$.

5. A bankrupt is indebted to A L. 120, to B L. 240, to C L. 350, and to D L. 460, and his whole estate amounts only to L. 680; how much must each creditor receive?

Ans. A *must receive* L. 69 14 $10\frac{18}{19}d$. B, L. 139 9 $8\frac{16}{19}d$. C, L. 203 8 $4\frac{2}{19}d$, *and* D, L. 267 7 $0\frac{4}{19}d$.

6. A and B venturing equal sums of money, cleared by joint trade L. 200; by agreement A, as he executed the business, was to have 9 *per cent.* and B 6 *per cent.*; what was A allowed for his trouble?

Ans. L. 40.

7. A ship, worth L. 1000, being lost at sea, of which $\frac{1}{7}$ belonged to A, $\frac{1}{2}$ to B, and the rest to C; what share of the loss will each sustain, supposing L. 400 to have been insured?

Ans. A's L. 120, B's, *L.* 300, *and* C's L. 180,

8. Four men spent, at a public house, $2\frac{1}{2}$ guineas; of which they agreed that A should pay $\frac{1}{4}$, B, $\frac{1}{2}$, C, $\frac{1}{4}$, and D, $\frac{1}{8}$; what must each pay in that proportion?

Ans. A, L. 1 12*s.* $3\frac{9}{11}$*d.* B, L. 1 1*s.* $6\frac{6}{11}$*d.* C, 10*s* $9\frac{3}{11}$*d.* *and* D, 5*s.* $4\frac{8}{11}$*d.*

9. A, B, and C put their money into a joint stock; A put in L. 160; B and C together, L. 680; they gained L. 504, of which C took L. 168; what did A and B gain, and B and C put in respectively?

Ans. A gained L. 96, and B, L. 240; B put in L. 400, and C, L. 280.

10. A, B, and C put in L. 810 and gained L. 420, of which, as often as A took L. 1, B took L. 3, and C 5; what did each put in and gain?

Ans. A's stock was L. 90, B's, L. 270, and C's, L. 450; and A's gain was L. $46\frac{2}{3}$, B's, L. 140, and C's, L. $233\frac{1}{3}$.

CASE 2.*

When neither the times nor stocks are equal.

Rule.

Multiply each man's stock, by the time it was continued in trade; then,

As the whole sum of the products is to the whole gain or loss; so is each man's particular product, to his particular share of the gain or loss.

Examples.

1. A and B hold a pasture in common; for which they are to pay L. 30. A put in 20 oxen for 10 weeks,

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* This *Case* is, what is usually called, though improperly, *Double Fellowship*; some examples, commonly arranged under which, may, and with more propriety, be wrought by *Single Fellowship*; such as examples *second* and *third*, in *Case first* above.

and B 25 oxen for 12 weeks; what ought each to pay for the rent?

$20 \times 10 = 200$
$25 \times 12 = 300$

Sum of the products, 500

L.		L.	
As 500	: 30 ::	200	: 12 *A's share of the rent.*
As 500	: 30 ::	300	: 18 *B's do.*
			30 *Proof.*

2. Two merchants entered into partnership for 18 months; A, at first, put into stock L. 200, and at the end of 5 months put in L. 40 more; B, at first, put in L. 300, and at 8 months end, he took out L. 100. At the expiration of the time, they found they had gained L. 400; what is each man's just share?

Ans. A's share is L. 193 8 6 $2\frac{10}{11}$*qrs.* and B's, L. 206 11 5 $1\frac{1}{11}$*qrs.*

3. Three persons joined in trade; A put in 700 dollars for 10 months; B, 800 dollars for 12 months, and C, 500 dollars for 15 months; but by misfortune, they lost goods to the value of 900 dollars; what must each person sustain of the loss?

Ans. A, 261 D. $41\frac{19}{141}$ cents, B, 358 D. 50 c. $6\frac{14}{141}$ m. and C, 280 D. 8 c. $2\frac{118}{141}$ m.

4. Four merchants entered into partnership for 20 months: A, at first, put into stock L. 200, and at 4 months end he put in L. 100 more, and 8 months after that, he put in L. 150 more. B put in, at first, L. 800; at the end of 5 months, he took out L. 200, and 10 months after, he took out L. 100 more. C, at first, put in L. 500; at the end of 6 months he took out L. 300, and 9 months after he put in L. 400. D put in, at first, L. 400, and at 8 months end, he put in L. 400 more, and 8 months after, he took out L. 200; at the expiration of the time, they found they had gained L. 1000; what is each man's share?

Ans. A L. 173 18 $3\frac{5}{19}\frac{1}{20}d.$ B L. 319 13 $10\frac{1}{19}\frac{1}{20}\frac{4}{20}d.$ C L. 199 9 $9\frac{9}{19}\frac{3}{20}d.$ and D L. 306 18 $1\frac{1}{19}\frac{1}{20}d.$

5. Two persons joined in trade; A put in, the first of January, L. 250; but B could not put in 'till the first of May; what did he then put in, to have an equal share with A at the year's end?

Ans. L. 375.

6. A, B, and C companied; A put in, the first of April, L. 40; B, the first of June, 12 *hhd.* of rum; and on the first of August, C put in 400 yards of broadcloth. On the first of February following, they divided their gains, of which A and B took L. 200; B and C, L. 400, and C and A, L. 210; what was the whole gain, and the gain of each; at what did they value B's rum per *hhd.* and what was C's broadcloth worth per yard?

Ans. the whole gain was L. 420, of which A's share was L. 20, B's, L. 180, and C's, L. 220; B's rum was valued at L. 37 10*s.* per *hhd.* and C's broadcloth at L. 1 16*s.* 8*d.* per yard.

ALLIGATION.

Alligation is the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality.

CASE 1.

From the quantities and prices of the several simples given, to find the rate of the compound.

RULE.

As the sum of the quantities, or the whole composi-

tion, is to their total value; so is any part of the composition to its mean price or value.

EXAMPLES.

1. Suppose 10 gallons of wine at 9*s.* per gallon, and 20 gallons, at 7*s.* 6*d.* per gallon, were mixed together; how must the compound be sold per gallon without gain or loss?

gal.
10 at 9*s.* is L. 4 10*s.*
20 at 7*s.* 6*d.* is L. 7 10*s.*

30 *Sum of the simp.* 12 *Total value.*

gal. *L.* *gal.*
30 : 12 :: 1 :
20

3,0) 24,0

8*s.* *the Ans.*

2. A composition being made of 6*lb.* of tea at $1\frac{1}{2}$ dollar *per lb.* 9*lb.* at 1 dollar, and 10*lb.* at 50 cents; what is a *lb.* of it worth? *Ans.* 92 *cents.*

3. A tobacconist mixed 40*lb.* of tobacco at 6*d.* *per lb.* with 30*lb.* at 9*d.* 20*lb.* at 1*s.* 6*d.* and 10 *lb.* at 2*s.*; what is 1*lb.* of this mixture worth?

Ans. $11\frac{1}{10}d.$

4. A farmer would mix 4 bushels of flour at 3*s.* 6*d.* per bushel, 5 bushels at 4*s.* 8 bushels at 4*s.* 9*d.* 10 bushels at 6s. and 15 bushels at 8*s.*; what is the value of a bushel of this mixture? *Ans.* 6*s.*

5. A goldsmith melted together 2*lb.* of gold of 22 carats fine, 1*lb.* 4*oz.* of 21 carats fine, 2*lb.* of 20 carats fine, and 2*lb.* 8*oz.* of 18 carats fine; what is the quality or fineness of the composition?

Ans. 20 *carats.*

CASE 2.

To find what quantity of each of the simples, whose rates are given, will compose a mixture of a given rate.

Rule 1.*

1. Write the rates of the simples, reduced to one denomination, in a column under one another, the least uppermost, and the mean rate at the left hand.

2. Connect, or link with a continued line, the rate of each simple, which is less than the mean rate, with one or any number of those which are greater, and each greater rate with one or any number of the less.

3. Place the difference between the mean rate and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate, bnt if there be more than one, their sum will be the quantity.

Examples.

1. How much wine at 9*s.* per gallon and at 7*s.* 6*d.* per gallon must be mixed together that the composition may be worth 8*s.* per gallon?

	d.	*d.*	*gal.*	
Mean rate	96	90	12	at 7*s.* 6*d. per gallon.*
		108	6	at 9*s. per gallon.*

* It is obvious, from the rule, that questions admit of a great variety of answers; *first*, by the different modes of connecting, or linking, and, *secondly*, by having found one answer, we may find as many more as we please, by only multiplying or dividing each of the quantities found by 2, 3, 4, &c.

If any of the simples be of little or no value with respect to the rest, its rate is not considered any thing; as water mixed with wine, and alloy with gold and silver.

Note, Case *second* is the reverse of Case *first*, and therefore, they prove each other.

2. How much corn at 3*s*. 6*d*. 4*s*. and 4*s*. 6*d*. per bushel, must be mixed together that the composition may be worth 3*s*. 10*d*. per bushel?

Ans. 10 *bushels at* 3*s*. 6*d*. 4 *at* 4*s*. *and* 4 *at* 4*s*. 6*d*.

3. A merchant has tea at 12*s*. 10*s*. 8*s*. and 6*s*. *per lb.*; how much of each must he mix together that the composition may be afforded at 9*s*. *per lb.*?

* Ans. 4*lb*. *of each sort.*

4. A goldsmith would mix gold of 18 carats fine, with some of 19, 20, 22, and 24 carats fine, so that the compound may be 21 carats fine; of how much of each sort must the mixture consist?

Ans. 4 *of* 18, 4 *of* 19, 4 *of* 20, 6 *of* 22, *and* 6 *of* 24 *carats fine.*

5. It is required to mix claret wine at 4*s*. sherry at 6*s*. and canary at 8*s*. per gallon, with water, that the composition may be worth 5*s*. per gallon; how much of each will it take?

Ans. 3 gallons of water, 1 gallon of claret wine at 4*s*. 1 gallon of sherry at 6*s*. and 3 gallons of canary at 8*s*.

6. How much alloy must be mixed with gold of 21, 22, 23, and 24 carats fine, that the compound may be 20 carats fine?

Ans. 10 of alloy with 20 of each sort of gold.

Rule 2.

When the whole composition is limited to a certain quantity.

Find an answer by Rule 1st. then say, as the sum of the quantities, or differences thus determined, is to the

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* Seven answers may be found to this question by as many different ways of linking the prices of the simples.

given quantity, or whole composition; so is each ingredient, found by linking, to the required quantity of each rate.

Examples.

1. A grocer has currants at 4*d.* and 10*d.* *per lb.* the worst would not sell and the best were too dear; he therefore concludes to mix 20*lb.* so as to sell them at 6*d.* *per lb.*; how much of each sort must he take?

	d.	*d.*	*lb.*
Mean rate	6	4	4
		10	2

Sum. 6

lb.	*lb.*					
As 6	: 20	::	4	:	$13\frac{1}{3}$ at 4*d.*	*per lb.*
			2	:	$6\frac{2}{3}$ at 10*d.*	

2. It is required to mix water with brandy at 6*s.* and 10*s.* per gallon, that 56 gallons of this mixture may be afforded at 8*s.* per gallon; how much water, and of each sort of brandy will it take?

Ans. 8 gallons of water, 8 gallons of brandy at 6*s.* per gallon, and 40 do. at 10*s.*

3. A grocer has sugar at 4*d.* 6*d.* 8*d.* and 10*d.* per *lb.* and he would have a composition of an *cwt.* worth 7*d.* *per lb.*; how much of each must he have?*

Ans. 28 *lb.* *of each.*

4. A goldsmith has 4 sorts of gold, viz. 23, 22, 21, and 20 carats fine, and he would mix with these so much alloy as that the quantity of 43*oz.* may bear 18 carats fine; how much of each sort, and how much alloy must he take?

Ans. 9 *oz.* of each sort of gold, and 7*oz.* of alloy.

---

* This queſtion admits of ſeven anſwers by as many ways of linking.

Rule 3.

When one of the simples, or ingredients is limited.

Find the difference between each price and the mean rate by Rule 1st. then say, as the difference standing against that simple, whose quantity is given, is to the given quantity; so is each of the other differences to the several quantities required.

Examples:

1. How much tea at 3*s.* and 6*s. per lb.* must be mixed with 28*lb.* at 12*s. per lb.* so that the mixture may be worth 8*s. per lb.* ?

	s.	*lb.*	*lb.*	
8*s.*	3	4	4	
	6	4	4	
	12	2+5	7	*Standing against the simple whose quantity is given.*

lb.	*lb.*		*lb.*	*lb.*	
As 7	: 28	::	4	: 16 at 3*s.*	*per lb.*
			4	: 16 at 6*s.*	

2. A farmer would mix 30 bushels of oats at 1*s.* 9*d.* per bushel, with barley at 3*s.* rye at 4*s.* and wheat at 6*s.* per bushel; how much barley, rye, and wheat must be mixed with the 30 bushels of oats, that the mixture may be worth 3*s.* 6*d.* per bushel.

**Ans.* 30 bushels of barley at 3*s.* per bushel, $22\frac{1}{2}$ bushels of rye at 4*s.* and $22\frac{1}{2}$ bushels of wheat at 6*s.*

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* Let the learner find all the anſwers that may be found by linking.

3. A merchant would mix 10*lb.* of spice at 6*s. per lb.* with some at 5*s.* some at 4*s.* some at 2*s.* and some at 1*s. per lb.*; how much of each must he mix with the 10*lb.* so as to sell the composition at 3*s. per lb.*?

Ans. 10*lb.* at 5*s. per lb.* 10*lb.* at 4*s.* 20*lb.* at 2*s.* and 20*lb.* at 1*s.*

4. How much gold of 16, 18, 22, and 23 carats fine, and how much alloy must be mixed with 6*oz.* of 24 carats fine, that the composition may be 20 carats fine?

Ans. $2\frac{1}{11}$ *oz.* of 16, $2\frac{1}{11}$ of 18, 6 of 22, and 6 of 23 carats fine, and $2\frac{1}{11}$ *oz.* of alloy.

Rule 4.

When two or more of the simples are limited.

Find, by Case 1st. what will be the *rate* of a mixture made of the *quantities* of the *limited simples*; then, consider this as the *rate* of a *limited simple*, whose *quantity* is the *sum* of the first *limited simples*, from *which* and the *rates* of the *unlimited simples* calculate, by Case 2d. Rule 3d. the quantity.

Examples.

1. How much tea at 7*s.* and 9*s. per lb.* must be mixed with 6*lb.* at 3*s.* and 10*lb.* at 5*s. per lb.* that the mixture may be worth 6*s. per lb.*?

	lb.				
Limited simples	6	at	3*s.*	=	18*s.*
	10	at	5*s.*	=	50*s.*
	16				68

lb. *s.* *lb.* *s.* *d.*

As 16 : 68 :: 1 : 4 3 *per lb.*

Now, having found the rate of the limited simples; the question may stand thus: How much tea at 7*s.* and 9*s. per lb.* must be mixed with 16*lb.* at 4*s.* 3*d. per lb.* that the mixture may be worth 6*s. per lb*?

$$72d. \left\{\begin{array}{l|l|l} d. & & \\ 51 & 36+12 & 48\ lb. \\ 84 & 21 & 21 \\ 108 & 21 & 21 \end{array}\right.$$

$$\text{As } \begin{array}{ll} lb. & lb. \\ 48 & 16 \end{array} :: \left\{\begin{array}{l} lb.\ :\ lb. \\ 21\ :\ 7 \text{ at } 7s. \\ 21\ :\ 7 \text{ at } 9s. \end{array}\right\} \textit{per lb.}$$

2. How much wine at 4*s.* and 5*s.* per gallon must be mixed with 4 gallons at 6*s.* 6 gallons at 8*s.* and 8 gallons at 9*s.* per gallon, that the composition may be worth 7*s.* per gallon?

Ans. $3\frac{1}{2}$ gallons at 4*s.* and $3\frac{1}{2}$ gallons at 5*s.* per gal.

3. How much spice at 8*s.* 7*s.* and 6*s.* *per lb.* must be mixed with 6*lb.* at 4*s.* 8*lb.* at 3*s.* and 10*lb* at 2*s.* *per lb.* that the mixture may be worth 5*s.* *per lb.*?

Ans. $8\frac{2}{3}$*lb.* of each sort.

4. A refiner would melt gold of 24, 23, 22, and 21 carats fine, with 8*oz.* of 20, 6*oz.* of 19, 4*oz.* of 18, and 2*oz.* of 17 carats fine; I demand how much of each sort of gold, and how much alloy he must mix with the 8*oz.* of 20, 6*oz.* of 19, 4*oz.* of 18, and 2*oz.* of 17 carats fine, that the composition may be 16 carats fine?

Ans. 20*oz.* of each sort of gold, and $36\frac{1}{4}$*oz.* of alloy.

INVOLUTION.

Involution is the finding of powers.

A *power* is a number produced by multiplying any number continually by itself a certain number of times.*

Note 1. The number, which exceeds the multiplications by 1, is called the *index* or *exponent* of the power; therefore, 1 is the index or exponent of the first power; 2 is the index of the second power, &c.

Note 2. Powers are generally denoted by writing their indices above the first power: so the second power of 4 may be expressed thus $4,^2$ the third power, thus $4,^3$ the fourth, thus $4,^4$ &c.

* Any number is the *first* power.

THE FIRST TEN POWERS OF THE 9 DIGITS.

Roots,	or 1st. Power.	1	2	3	4	5	6	7	8	9
Squares,	or 2d. Powers.	1	4	9	16	25	36	49	64	81
Cubes,	or 3d. Powers.	1	8	27	64	125	216	343	512	729
Biquadrates,	or 4th. Powers.	1	16	81	256	625	1296	2401	4096	6561
Sursolids,	or 5th. Powers.	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes,	or 6th. Powers,	1	64	729	4096	15625	46656	117649	262144	531441
Second Sursolids.	or 7th. Powers.	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates Squared,	or 8th. Powers.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes Cubed,	or 9th. Powers.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Sursolids Squared,	or 10th. Powers.	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401

To raise Powers.

RULE.

Multiply the given number, or first power, continually by itself, 'till the number of multiplications be 1 less than the index of the required power, and the last product will be the answer.*

NOTE. Fractions are involved by raising each of their terms to the power required; and if a mixed number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and then proceed by the rule.

EXAMPLES.

1. What is the second power of 24?

24 = 1*st. Power.*
24
—
96
48
——
576 = 2*d. Power, or Ans.*

2. What is the second power, or square of 120?
Ans. 14400.

3. What is the third power of 4·5?
Ans. 91·125.

4. What is the biquadrate or fourth power of $\frac{4}{7}$?
Ans. $\frac{256}{2401}$.

---

* Whatever two or more powers are multiplied together, their product is the power whose index is the sum of the indices of the factors; hence, a power may sometimes be found more concisely than by the general Rule above, thus; if you would find the fourth power, you may, first, find the second, then multiply the second by itself, and the product will be the fourth; or if you would find the seventh power, you may find the third and fourth, and their product will be the seventh, &c.

EVOLUTION.

Evolution is the extraction of the roots of numbers.

A *root* is a number, whose continual multiplication into itself a certain number of times produces the power, and is denominated the *second, third, fourth, &c.* root, or the *square, cube, biquadrate, &c.* root, according to the *index* or *exponent* of the power.

NOTE 1. The *index* of the root is 1 more than the number of multiplications necessary to produce the power.

NOTE 2. A number is called a complete power, when its root can be accurately extracted; but if not, the number is denominated an imperfect power, and its root a surd or irrational number; so 16 is a complete power or square, its root being 4: but 16 is an incomplete or imperfect power or cube, and its root a surd number.

NOTE 3. A power is prepared for evolution or extraction by dividing it from the place of units to the left hand in integers, and to the right in decimals, into periods, each containing as many places of figures, when they are complete, as are denoted by the index of the root; but if all the periods of a power be not complete, the defect will be, either on the right, or left, or both; if it be on the right, the deficiency may be supplied by annexing cyphers, and afterwards whole periods of cyphers may be annexed and the extraction may be continual as far as necessary; but if the defect be on the left, the defective period must remain unaltered, and be numbered a period as though it were complete.

Now, this division may be conveniently performed by making a point over the place of units, and also over the first figure of every period on each side of it; *i. e.* over every second figure beyond the place of units for the square root, over every third figure, for the cube root, &c.

Thus, to point this number 87654321·23456.

First, for the *second* root, 87654321·234560.

Secondly, for the *third*, 87654321·234560.

Thirdly, for the *fourth*, 87654321·23456000.

NOTE 4. The root will contain just as many places of figures, both of integers and decimals, as there are periods or points of each in the given power.

THE EXTRACTION OF THE SQUARE ROOT.

RULE.

1. Having distinguished the given number into periods, find, by the table or trial, the greatest square number in the left hand period, and write the root of it on the right hand of the given number, after the manner of a quotient in division, for the first figure of the root.

2. Subtract the assumed square from the left hand period, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. Consider how many times the divisor is contained in the dividend, exclusive of the place of units, and place the result both in the root and on the right hand of the divisor.

5. Subtract the product of the divisor, thus augmented, and the last figure in the root, from the dividend, and to the remainder bring down the next period for a new dividend.

6. Double the figures of the root for a new divisor (or bring down the last divisor, doubling the right hand figure of it, for a new one) by which find the next figure

of the root as before, and thus proceed till every period is brought down.

NOTE 1. When the number to be extracted is not a complete power, the work may be abbreviated thus: having proceeded in the extraction by the rule till 1 more than half of the decimal part of the root is found, the rest may be found by dividing the last remainder by its corresponding divisor as in division of decimals.

NOTE 2. The method of proof is, to multiply the root by itself, adding the remainder, if there be any, and the sum will be equal to the given number, when the work is right.

NOTE 3. The most convenient way of doubling the root is, by adding the last figure of it to the last divisor.

NOTE 4. By the square root, we may find the root of any power, whose index is a power of the number 2; so the fourth root, the eighth root, the sixteenth root, &c. may be readily found, namely, by extracting so often the square root, as is denoted by the power of 2, that is, the fourth root is found by extracting the square root twice, the eighth root, by extracting thrice, &c.

RULES FOR THE SQUARE ROOT OF VULGAR FRACTIONS AND MIXED NUMBERS.

Prepare the vulgar fraction for extraction by reducing it to its lowest terms, for this and all other roots; then,

1. Extract the roots of the numerator and denominator for the respective terms of the root required; which is the best method, if the denominator be a complete power. But if not, then,

2. Multiply the numerator and denominator together; then make the root of this product, the numerator to the

denominator of the given fraction, or make the root, the denominator to the numerator of it; and it will form the fractional root required.

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixed numbers must be either reduced to improper fractions and extracted by the first or second rule; or the vulgar fraction must be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

EXAMPLES.

1. What is the square root of 1522756?

```
                1522756(1234 the root or Ans.
                1
              ---------
              22 | 52
               2 | 44
              ---------
              243 | 827
                3 | 729
              ---------
              2464 | 9856
                   | 9856
                   ------
                   .... 1234
                        1234
                        ----
                        4936
                       3702
                      14808
                      -----
                      1522756 Proof.
                      -------
```

2. Required the square root of 550183936.

Ans. 23456.

3. What is the square root of 428135971041?

Ans. 654321.

4. What is the square root of ·000018671041 ?
Ans. ·004321.

5. Required the square root of 16501875·0625.
Ans. 4062·25.

6. What is the square root of 2 ?
Ans. 1.414213562, &c.

7. What is the square root of 1606246516·125 ?
Ans. 40078·00539, &c.

8. What is the square root of $\frac{875}{5040}$? Ans. $\frac{5}{12}$.

9. What is the square root of $\frac{4936}{10850}$? Ans. $\frac{2}{5}$.

10. What is the square root of $72\frac{1}{4}$? Ans. $8\frac{1}{2}$.

EXTRACTION OF THE CUBE ROOT.

Rule.

1. Having divided the given number into periods of three figures each ; find the greatest cube in the left hand period ; subtract it there from, placing the root in the quotient, and to the remainder bring down the next period and call it the *dividend.*

2. Under the dividend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former ; and under the triple square write the triple root, removed one place to the right hand, and call their sum the *divisor.*

3. Seek how often the divisor may be had in the dividend, exclusive of the place of units, and write the result in the quotient.

4. Under the divisor write the product of the triple square of the root by the last quotient figure, placing the unit's figure of the product under the place of tens in the divisor ; under this line, write the product of the triple root by the square of the last quotient figure, removed one place to the right hand of the former ; and, under this line, removed one place to the right hand, write the cube of the last quotient figure, and call their sum the *subtrahend.*

5. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on through all the periods.

Examples.

1. Required the cube root of 12812904.

12812904(234 *Root.*
8

4812	*first dividend.*
12	*triple square of* 2.
6	*triple of* 2.
126	*first divisor.*
36	*triple square of* 2 *multiplied by* 3.
54	*triple of* 2 *multiplied by the square of* 3.
27	*cube of* 3.
4167	*first subtrahend.*
645904	*second dividend.*
1587	*triple square of* 23.
69	*triple of* 23.
15939	*second divisor.*
6348	*triple square of* 23 *multiplied by* 4.
1104	*triple of* 23 *multiplied by the square of* 4.
64	*cube of* 4.
645904	*second subtrahend.*

2. What is the cube root of 80621568 ?
Ans. 432.

3. What is the cube root of 224755712 ?
Ans. 608.

4. What is the cube root of 1879080·904 ?
Ans. 123·4.

5. Required the cube root of ·008646682203125.
Ans. ·20525.

6. What is the cube root of $\frac{1}{3\,3\,7\,4}$? Ans. $\frac{1}{7}$.

7. What is the cube root of $\frac{6\,2\,5}{2\,5\,0\,0}$? Ans. $\frac{5}{6}$.

8. What is the cube root of $1157\frac{5}{8}$? Ans. $10\frac{1}{2}$.

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

Rule.

1. Divide the given number into periods according to the index of the root required.

2. Find the first figure of the root, by the table of powers or by trial, and subtract its power from the given number.

3. To the remainder bring down the first figure in the next period, and call it the *dividend.*

4. Involve the root to the next inferior power to that which is given, and multiply it by the index of the root required, for a *divisor.*

5. Find how often the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Raise the whole root to the given power, and subtract it from the given number as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which, find a new divisor as before, and so on, 'till the whole be completed.

Examples.

1. What is the cube root of 33076161 ?

33076161(321
27 = $3.^3$

$3^2 \times 3 = 27)60 =$ *dividend.*

32768 = $32.^3$

$32^2 \times 3 - 3072)\ 3081 =$ 2d. *dividend.*

33076161 = $321.^3$

2. What is the biquadrate root of 4100625 ?
Ans. 45.
3. What is the biquadrate root of 13397430·0625 ?
Ans. 60·5.
4. Extract the sursolid root of 1258284197543.
Ans. 263.
5. What is the square cubed, or sixth root of 353094 5043777457216 ? Ans. 1234.
6. Find the second sursolid, or seventh root of 201870 6186603943746202655390625. Ans. 21345.

ARITHMETICAL PROGRESSION.

Any rank of numbers, increasing by a common excess, or decreasing by a common difference, is said to be in *Arithmetical Progression;* as, 1, 2, 3, 4, &c. 4, 3, 2, 1; and ·9, ·8, ·7, ·6, &c. The former rank is cal-

led an *ascending series*, or progression; but the latter, a *descending one*.

The numbers, which form the series, are called the *terms* of the progression.

Any *three* of the five following terms being given, the other two may be easily found.

1. The *first* term.*
2. The *last* term.
3. The *number* of terms.
4. The *common difference*.
5. The *sum* of all the terms.

PROBLEM 1.

When the first term, the last term, and the number of terms are given, to find the sum of all the terms.

RULE.

Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. The extremes of an arithmetical progression are 1 and 39, and the number of terms, 20; what is the sum of the series?

$$\begin{array}{r} 39 \\ +\ 1 \end{array}\Big\} \textit{Extremes.}$$

$$\begin{array}{rl} 40 & \textit{Sum.} \\ \times 20 & \textit{the number of terms.} \\ \hline 2)800 & \\ \hline 400 & \textit{The Ans.} \end{array}$$

Or, $\frac{39+1\times 20}{2}=400.$

* The first and last terms of a progression are usually called the *extremes*, and the other terms the *means*.

2. How often will the hammer of a repeating clock strike in 24 hours? Ans. 300.

3. What is the sum of the first 100 numbers in their natural order? Ans. 5050.

4. If 1000 stones be placed in a right line, exactly one yard from each other, and the first, a yard from a basket; what length of ground must that man travel over, who gathers them up singly, returning with them one by one to the basket?

Ans. $568\frac{1}{4}$ *miles.*

PROBLEM 7.

The first term, the last term, and the number of terms being given, to find the common difference.

Rule.

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference required.

Examples.

1. The extremes are 1 and 39, and the number of terms 20; required the common difference?

20 39
1 1

19)38(2
38
—

Or, $\frac{39-1}{20-1}=2$ *the Ans.*

2. A man is to travel from Portsmouth to a certain place in 18 days, and to go but 3 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 71 miles; required the daily increase, and the distance of the place from Portsmouth.

Ans. daily increase 4, *and distance* 666 *miles.*

PROBLEM 3.

Given the first term, the last term, and the common difference, to find the number of terms.

Rule.*

Divide the difference of the extremes by the common difference, and the quotient, increased by 1, is the number of terms required.

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* *The following Table contains a summary of the whole doctrine of Arithmetical Progression.*

Cases of Arithmetical Progression.

Case	Giv.	Req.	Solution.
1.	*aln.*	d	$\frac{l-a}{n-1}$
		s	$\frac{\overline{a+l}\times n}{2}$
2.	*ald.*	n	$\frac{l-a}{d}+1$
		s	$\frac{\overline{l+a}\times\overline{\overline{l-a}+d}}{2d}$
3.	*als*	d	$\frac{\overline{l+a}\times\overline{l-a}}{2s.-\overline{l+a}}$
		n	$\frac{2s}{a+l}$

EXAMPLES.

1. The extremes are 1 and 39, and the common difference 2; what is the number of terms ?

Case	Giv.	Req.	Solution.
4.	adn	l	$\sqrt{n-1 \times d+a}$
		s	$\sqrt{n \times a+n-1 \times \frac{d}{2}}$
5.	$ans.$	d	$\frac{2 \times \sqrt{s-an}}{n-1 \times n}$
		l	$\frac{2s}{a}-a$
6.	lns	a	$\frac{2s}{n}-l$
		d	$\frac{2 \times \sqrt{nl-s}}{n-1 \times n}$
7.	lnd	a	$l-\sqrt{n-1 \times d}$
		s	$\sqrt{n \times l-\sqrt{n-1 \times \frac{d}{2}}}$

$$\left.\begin{matrix}39\\1\end{matrix}\right\}\ \textit{Extremes.}$$

Common difference $2)38$

Quotient 19

Add 1

20 *the Ans.*

Or, $\frac{39-1}{2}+1=20.$

Case	Giv.	Req.	Solution.
8.	dns	a	$\frac{s}{n}-\frac{d\times\overline{n-1}}{2}$
		l	$\frac{s}{n}+\frac{d\times\overline{n-1}}{2}$
9.	ads	n	$\frac{\sqrt{\overline{2a-d}^2+8ds}-\overline{2a-d}}{2d}$
		l	$\frac{\sqrt{\overline{2a-d}^2+8ds}-d}{2}$
10.	dls	a	$\frac{d\pm\sqrt{\overline{2l+d}^2-8ds}}{2}$
		n	$\frac{2l+d\pm\sqrt{\overline{2l+d}^2-8ds}}{2d}$

Here
- a = first term
- l = last term
- n = number of terms
- d = common difference
- s = sum of all the terms.

2. A man, going a journey, travelled the first day 7 miles, the last day 67 miles, each day increased his journey 5 miles; how many days did he travel, and how far?

Ans. 13 *days, and* 481 *miles.*

GEOMETRICAL PROGRESSION.

Geometrical Progression is, when any rank or series of numbers increases or decreases by the continual multiplication or division of the same number.*

PROBLEM 1.

Given the first term, the last term, and the ratio, to find the sum of the series.

Rule.

Multiply the last term by the ratio; from the product subtract the first term, and the remainder, divided by the ratio less 1, will be the answer.

Examples.

1. The first term of a geometrical series is 1, the last term 2048, and the ratio 2; required the sum of all the terms.

---

* The number, by which the series is increased or diminished, is called the *ratio.*

$$\begin{array}{r} 2048 \\ 2 \\ \hline 4096 \\ 1 \\ \hline 2-1=1)4095 \\ \hline 4095 \end{array}$$

$$\textit{Or.}\quad \frac{2048\times 2-1}{2-1}=4095 \textit{ the Ans.}$$

2. The first term of a series in geometrical progression is 3, the last term 14348907, and the ratio 3; what is the sum of the series? Ans. 21523359.

PROBLEM 2.

Given the first term and the ratio, to find any other term assigned.

RULE.*

1. Write down a few of the leading terms of the series, and place their indices over them, beginning with a cypher.

* *The following Table contains all the possible Cases in Geometrical Progression.*

CASES OF GEOMETRICAL PROGRESSION.			
Case	Giv.	Req.	Solution.
1.	arn	l	ar^{n-1}
		s	$\frac{r^{n}-1}{r-1}\times a$

2. Add together the most convenient indices to make an index less by 1 than the number expressing the place of the term sought.

Case	Giv.	Req.	Solution.
2.	arl	s	$l + \frac{l-a}{r-1}$
		n	$\frac{L.\,l - L.\,a}{L.\,r} + 1$
3.	ars	l	$\frac{\overline{r-1} \times \overline{s+a}}{r}$
		n	$\frac{L.\,\overline{r-1} \times \overline{s+a} - L.\,a}{L.\,r}$
4.	asl	r	$\frac{s-a}{s-l}$
		n	$\frac{L.\,l - L.\,a}{L.\,\overline{s-a} - L.\,\overline{s-l}} + 1$
5.	ans	r	
		l	

3. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power whose index is 1 less than the number of terms multiplied, and make the result a divisor.

Case	Giv.	Req.	Solution.
6.	*anl*	*r*	$\sqrt[n-1]{\frac{l}{a}}$
		s	$l + \frac{l-a}{\sqrt[n-1]{\frac{l}{a}} - 1}$
7.	*rnl*	*a*	$\frac{l}{r^{n-1}}$
		s	$l + \frac{l - \frac{l}{r^n - 1}}{r - 1}$
8.	*rns*	*a*	$\frac{r-1}{r^n - 1} \times s$
		l	$\frac{r^n - r^{n-1}}{r^n - 1} \times s$

5. Divide the dividend by the divisor, and the quotient will be the term sought.

NOTE, When the first term of the series is equal to the ratio, the indices must begin with an unit, and the indices added must make the entire index of the term required; then the continued product of the terms, whose indices are added, will give the answer required.

EXAMPLES.

1. The first term of a geometrical progression is 2 and the ratio 3; what is the 12th. term?

Case	Giv.	Req.	Solution.
9.	*rls*	*a*	$s - r \times \overline{s - l}$
		n	$\dfrac{L.\,l - L.\,\overline{s - r \times \overline{s - l}}}{L.\,r} + 1$
10.	*nls*	*a*	
		r	

Here
- a = First or least term.
- l = Last or greatest term.
- s = Sum of all the terms.
- n = Number of Terms.
- r = Ratio.
- L = Logarithm.

0, 1, 2, 3, 4, 5, *indices.*
2, 6, 18, 54, 162, 486, *leading terms.*
Then 5+4+2=*index to the* 12*th. term.*
And 486×162×18=1417176=*dividend.*

The number of terms multiplied is 3, *and* 3—1=2 *is the index of the power to which the first term* 2 *is to be raised; therefore* 1417176÷4=354294 *the* 12*th. term required.*

2. The first term of a geometrical series is 2, and the ratio 2; required the 14th. term.

Ans. 16384.

3. Required the 16th. term of a rank of numbers in geometrical progression, the first term of which is 3, and ratio 4. Ans. 3221225472.

4. The first term of a geometrical series is 4, the ratio 2, and the number of terms 20; what is the last term?

Ans. 2097152.

QUESTIONS

TO BE ANSWERED BY THE TWO PRECEDING PROBLEMS.

1. A merchant sold 18 yards of broad cloth on the following terms, *viz.* 2d. for the first yard, 6d. for the second, 18*d.* for the third, and so on, in triple proportion; I demand the price of the broadcloth.

Ans. L. 1614252 0s. 8d.

2. What debt will be discharged in 12 months, by paying 1 dime the first month, 5 the second, &c. each succeeding payment being 5 times the last; and what will the last payment be?

Ans. *the debt is* 6103515 *dollars,* 60 *cents, and the last payment* 4882812½ *dollars.*

3. A young man, well skilled in numbers, agreed with a rich farmer to work for him 8 years, only for 5 cents the first year, 50 the second, and so on, in a ten fold proportion; what did his 8 years' labour amount to, and what did it cost the farmer a year?

Ans. *his* 8 *years' work amounted to* 555555 *dollars*, 55 *cents*, *which cost the farmer* 69444 *dollars*, 44$\frac{4}{9}$ *cents a year.*

4. A butcher wanting to purchase 40 sheep for the market, a gentleman farmer told him that he would let him have the complement on these moderate terms, *viz.* that he should give him only 1 mill for the first sheep, 2 for the second, 4 for the third, &c. doubling at every sheep; to which the butcher, thinking to make a very good bargain, readily agreed: what did the sheep cost the butcher, and how much did the last sheep come to?

Ans. *the sheep cost the butcher* 1099511627 *dollars*, 77$\frac{1}{2}$ *cents*, *and the last sheep came to* 549755813 *dollars*, 88 *cents*, 8 *mills.*

INTEREST.

Interest is a premium for the loan of money, at any rate *per cent. per Annum* agreed on.

Principal is the money lent.

Rate or *Ratio* is the sum *per cent.* agreed on.

Amount is the sum of the principal and interest.

Interest is of two kinds, *simple* and *compound.*

SIMPLE INTEREST.*

Simple Interest is that, which is allowed only for the principal lent.

* Commission, Brokerage, Insurance, Stocks, and, in general, whatever is rated at so much *per cent.* are calculated by Simple Interest.

Rule.*

1. Multiply the principal by the ratio, divide the product by 100, and the result will be the interest for one year.

2. Multiply the interest for 1 year by the number of years, and the product will be the answer for that time.

3. If there be parts of a year, as months or days, work by practice or simple proportion.

Examples.

I. What is the interest of L. 350 10*s*. 5*d*. for 4½ years, at 6 *per cent. per Annum ?*

L.	*s.*	*d.*	
350	10	5	
		6	
21·03	2	6	
20			
0·62			
12			
7·50			
4			
2·00			

L.	*s.*	*d.*	
21	0	7½	*Interest for* 1 *year.*
		4½	
84	2	6	*Do. for* 4 *years.*
10	10	3¾	*Do. for* ½ *year.*
94	12	9¾	*The Answer.*

* The Rule is evidently an application of Simple Proportion and Practice.

2. What is the interest of L. 79 15*s*. for 1 year, at 6 *per cent.*? Ans. L. 4 15 8 $1\frac{1}{5}$.

3. What is the interest of L. 98 9*s*. 6*d*. for 3 years, at 6 *per cent.*? Ans. L. 17 14*s*. $6\frac{3}{5}$*d*.

4. What is the interest of L. 119 for 4 years, at $4\frac{1}{2}$ per cent.? Ans. L. 21 8*s*. $4\frac{4}{5}$*d*.

5. What is the interest of L. 120 for 3 months, at 6 per cent.? Ans. L. 1 16*s*.

6. What is the interest of L. 300 for 7 months, at 8 per cent.? Ans. L. 14.

7. What is the interest of L. 400 19*s*. 9*d*. for 14 months, at 6 per cent.? Ans. L. 28 1 4 $2\frac{9}{5}$.

8. What is the interest of L. 800 for 16 months, at 9 per cent. per Annum? Ans. L. 96.

9. Required the interest of L. 1409 10*s*. for 20 months, at 3 per cent. per Annum.

Ans. L. 70 9*s*. 6*d*.

Twelve months, in calculating interest, are allowed to a year; therefore, when the rate is 6 *per cent.* multiply the principal by one half the given number of months, then divide the product according to the rule, and the quotient will be the interest for the whole number of months. Hence,

When the rate *per cent. is* { 9, 8, 6, 4, 3, 2, $1\frac{1}{2}$, 1 } multiply the principal by { $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{12}$ } of the given number of months, then divide the product as the rule directs, and you will have the answer for the time.

10. What is the amount of L. 240 for 21 months, at 4 per cent. per Annum? Ans. L. 256 16.

11. What is the amount of L. 348 16*s*. 4*d*. for $8\frac{3}{4}$ years, at $5\frac{3}{4}$ per cent. per Annum?

Ans. L. 524 6 3 $2\frac{9}{20}qrs.$

12. What is the amount of L. 685 for 16 years, 11 months, and 21 days, at 6 per cent. per Annum?

Ans. L. 1382 13 $5\frac{2}{7}d.$

SIMPLE INTEREST BY DECIMALS.

RULE.*

Multiply together the principal, ratio,† and time, and it will give the interest required.

---

* The following Theorems will exhibit all the possible Cases of simple Interest, where p=principal, r=ratio, t=time, and a=amount.

1. $prt+p=a$. 2. $\frac{a}{rt+1}=p$.

3. $\frac{a-p}{pt}=r$. 4. $\frac{a-p}{pr}=t$.

† Ratio, in decimals, is the simple interest of L. 1. or 1 dollar, at the rate *per cent.* agreed on.

Thus, the ratio, at { 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$ } *per cent.* is { ·02, ·025, ·03, ·035, ·04, ·045, ·05, ·055, ·06, ·065 }

Examples.

1. What is the interest of L. 825 10*s.* for 2 years, at 6 per cent. per Annum ?

825·5
·06

49·530
2

99·060
20

1·200
12

2·400

Ans. L. 99 1*s.* $2\frac{2}{5}$*d.*

2. What is the interest of L. 125 15s. for 6 years, at $5\frac{1}{2}$ per cent. per Annum ?

Ans. L. 41 9*s.* $11\frac{7}{1}$*d.*

3. What is the interest of 227 dollars for 3 years, at 5 per cent. ? Ans. 34 dollars and 5 cents.

4. Required the interest of 435 dollars and 25 cents, at 6 per cent. for 4 years. Ans. 104·46.

5. What is the interest of 984 dollars and 5 dimes for $7\frac{1}{2}$ years, at 6 per cent. ?

Ans. 443 dollars, 2 c. 5 m.

6. What is the interest of $548\frac{3}{4}$ dollars for 10 years, at $6\frac{1}{2}$ per cent. per Annum ?

Ans. 356 dollars, 68 cents, and $7\frac{1}{2}$ mills.

7. What is the amount of 400 dollars and 87 cents for $5\frac{3}{4}$ years, at 6 per cent. per Annum ?

Ans. 539·170$\frac{3}{10}$ Dollars.

To calculate interest for days.

RULE 1.

Multiply continually the principal, ratio, and time; then divide by 365, and the result is the answer.

EXAMPLES.

1. What is the interest of L.345 14*s.* for 73 days, at 6 per cent. per Annum?

$$\frac{345{\cdot}7 \times {\cdot}06 \times 73}{365} = \text{L.4 2 } 11\tfrac{77}{365}d \text{ } \textit{Ans.}$$

2. What is the amount of 200 dollars for 292 days, at 6 per cent.? Ans. 209 D. 60 cents.

RULE 2.*

When the rate is 5 *per cent.* Multiply the principal by the given number of days; divide the product by 7300, and the quotient is the answer.

EXAMPLE.

What is the interest of 465 dollars and 45 cents for 300 days, at 5 per cent. per Annum?

$$\frac{465{\cdot}45 \times 300}{7300} = 19 \text{ D. } 1 \text{ d. } 2 \text{ c. } 8\tfrac{6}{73} \text{ m. } \textit{the Answer.}$$

RULE 3.

When the rate is 6 *per cent. per annum.*

Find the interest at 5 *per cent.* by the last rule; then add one fifth of the result to itself, and the sum is the answer.

---

* 7300 is the number of days in which any sum, at 5 *per cent. per annum*, will double; and it is found thus,

L.	*d.*		*L.*	*d.*
5	: 365	::	100	: 7300.

EXAMPLE.

What is the interest of 219 dollars for 146 days, at 6 per cent. per Annum ?

```
  219
  146
 ----
 1314
3066
----      5
73|00)319|74(4·38
  292      ·876
 ----    ------
  277    5·256 the Ans.
  219
 ----
  584
  584
 ----
  ...
```

DISCOUNT.

Discount is an allowance made for the payment of any sum of money before it becomes due; and is the difference between that sum due some time hence, and its present worth.

The *present worth* of any sum or debt due some time hence is such a sum, as, if put to interest, would, in the time and at the rate *per cent.* for which the discount is to be made, amount to the sum or debt, then due.

RULE 1.

1. As the *amount* of L. 100 for the given rate and time is to L. 100; so is the given sum or debt to the present worth.

2. Subtract the *present worth* from the given sum, and the remainder is the discount required.

RULE 2.

As the amount of L.100 for the given rate and time is to the interest of L.100 for the same time; so is the given sum or debt to the discount required.

EXAMPLES.

1. What is the discount of L. 413 15*s.* due 2 year hence, at 5 *per cent.* per annum ?

By Rule 1.

*Int. of L.*100 *per annum.* } = *L.*5
2 years.

10
Add 100

As L.110 : 100 :: 413 15
20 20

2200 8275
100

22|00)8275|00(376 *L.*
66

167
154

135
132

3
20

22)60(2*s.*
44

16
12

22)192(8$\frac{8}{11}$*d.*
176

16.

L. 413 15 *Given sum.*
L. 376 2 8$\frac{8}{11}$ *Present worth.*

37 12 3$\frac{3}{11}$ *Discount or Ans.*

By Rule 2.

L.	L.	L. s.
As 110	: 10	: : 413 15
20		20
2200		8275
		10
		L.
	22|00)827|50(37	
	66	
	167	
	154	
	135	
	20	
	22|0)270|0(12*s.*	
	22	
	50	
	44	
	6	
	12	
	22)72(3$\frac{3}{11}$*d.*	
	66	
	6	

Ans. L. 37 12*s.* 3$\frac{3}{11}$*d. the discount required.*

2. What is the discount of L. 800 which is due 3 years hence, at 6 *per cent. per annum?*

Ans. L. 122. 0*s.* 8$\frac{8}{17}$*d.*

3. What is the present worth of L. 240 due 4 years hence, at 6$\frac{1}{2}$ *per cent. per Annum?*

Ans. L. 190 9*s.* 6$\frac{6}{17}$*d.*

4. Required the ready money, which will discharge a debt of L.450 due 4 years and 7 months hence, at 6 *per cent. per Annum ?*

Ans. L. 352 18*s*. 9*d*. $3\frac{2}{3}\frac{7}{1}qr$.

DISCOUNT BY DECIMALS.

RULE.*

As the amount of L. 1 for the given rate and time is to the interest of L. 1 for the same time ; so is the given sum or debt to the discount required.

NOTE. Subtract the discount from the given sum and the remainder is the present worth.

EXAMPLES.

1. What is the discount of L. 700, due 2 years hence, at 6 *per cent. per Annum ?*

·06×2+1=1·12=*amount of* L. 1 *for the given rate and time.*

L.	L.	L.
As 1·12 :	·12 ::	700 :
	700	

1·12)84·00(75 L. *the discount required.*

784

560

560

...

* Let a=amount of any debt, p=present worth, r=ratio, and t=time; then will the following *Theorems* exhibit all the possible *Cases* in discount at simple interest.

I. $\frac{a}{rt+1}=p.$ II. $prt+p=a.$

III. $\frac{a-p}{pt}=r.$ IV. $\frac{a-p}{pr}=t.$

2. What is the discount of 360½ dollars, due 4 years hence, at 6 per cent. per annum?

Ans. 69 D. 77 c. $4\frac{6}{11}$m.

3. What is the present worth of 930¼ dollars, due 3 years and 10 months hence, at 6¼ per cent.?

Ans. 739·026$\frac{8\,1\,8}{1\,0\,0\,7}$ D.

4. How much money will discharge a debt of 2000 dollars, which is due 2 years, 5 months, and 18 days hence, discounting at 6 per cent.?

Ans. 1742 D. $16\frac{5}{1\,0\,7}$ c.

COMMISSION.

Commission is an allowance of so much *per cent.* to a factor or correspondent abroad, for buying and selling goods for his employer.

EXAMPLES.

1. What comes the commission on L. 609 18 4*d.* to, at 2½ per cent.?

L.	*s.*	*d.*
609	18	4
		2½
1219	16	8
304	19	2
15·24	15	10
20		
4·95		
12		
11·50		
4		
2·00		

Ans. L. 15 4*s.* 11½*d.*

2. What must I allow my factor for disbursing on my account *L.* 450 10*s.* at $2\frac{1}{4}$ per cent. ?

Ans. *L.*12 7 9 $1\frac{1}{2}$*qr.*

3. My correspondent receives 2400 dollars to lay out, after deducting his commission at $3\frac{5}{8}$ per cent. , what does his commission amount to ?

* Ans. 83 D. 9 d. 5 c. $6\frac{476}{849}$ m.

BROKERAGE.

Brokerage is an allowance of so much *per cent.* to an agent called a broker, for assisting merchants or factors in purchasing or disposing of goods.

EXAMPLES.

1. What is the brokerage of *L.* 535 15 9*d.* at 15*s.* or L.$\frac{3}{4}$ per cent. ?

s.		*L.*	*s.*	*d.*
10	$\frac{1}{2}$	535	15	9
5	$\frac{1}{2}$	267	17	$10\frac{1}{2}$
		133	18	$11\frac{1}{4}$
		4·01	16	$9\frac{1}{4}$
		20		
		0·36		
		12		
		4·41		
		4		
		1·67		

Ans. L. 4 0 4 1 · 67 *qr.*

* Since his commission is to be deducted from the money which he received from me, it is evident that I ought not to pay him com-

2. What is the brokerage of L. 800 at $\frac{7}{8}$ per cent. ?
Ans. L. 7.

3. If a broker be allowed $1\frac{1}{4}$ per cent.; what is his demand for selling goods to the amount of 1200 dollars ? Ans. 15 dollars.

BUYING AND SELLING OF STOCKS.

Stock is a general name for public funds, and the buying and selling of certain sums of money in those funds is not unusual.

EXAMPLES.

1. What is the perchase of L. 300 bank stock, at $84\frac{1}{2}$ per cent. ?

L.		*L.*
10	$\frac{1}{10}$	300
5	$\frac{1}{2}$	30
$\frac{1}{2}$	$\frac{1}{10}$	15
		1 10
	Subtract	46 10
		L. 253 10 *the Answer.*

2. What is the purchase of L. 1000 bank stock at 94 per cent. ? Ans. L. 940.

---

miſſion on his own money, which, however, is ſometimes unjuſtly practiſed; therefore,

D. *D.* *D.* *D.*
As 103·625 : 3·625 :: 2100 : 83·956$\frac{4}{8}\frac{5}{9}$.

3. What is the purchase of L. 2500 in the Portsmouth union bank at L. $120\frac{1}{4}$ per cent.?

Ans. L. 3018 15.

4. What does 15000 dollars capital stock, in Salem bank, amount to, at $125\frac{1}{8}$ per cent.?

Ans. $18806\frac{1}{4}$ dollars.

INSURANCE.

Insurance is an allowance of so much *per cent.*, made to certain persons who engage to make good the loss of houses, ships, *&c.* which may happen by fire, *&c.*

Examples.

1. What is the insurance of 3000 dollars, at $13\frac{1}{2}$ per cent.?

D.		D.
10	$\frac{1}{10}$	3000
2	$\frac{1}{5}$	300
1	$\frac{1}{10}$	60
$\frac{1}{2}$	$\frac{1}{2}$	30
		15
		405

Or thus,

3000
$13\frac{1}{2}$

39000
1500

405·00

Ans. 405 dollars.

2. What is the insurance of 4500 dollars, at $20\frac{1}{4}$ per cent.? Ans. 933 D. 75 c.

3. What is the insurance of 12000 dollars, at 30 per cent.? Ans. 3600 dollars.

4. What is the insurance of a ship and cargo, valued at 240000 dollars, at $45\frac{7}{8}$ per cent.?

Ans. 110100 dollars.

COMPOUND INTEREST.

Compound Interest is that, which arises from the interest being added to the principal, and becoming a part thereof, at the end of each year.

RULE.

1. Find the amount of the given principal, for the first year, by simple interest; then this amount will be the principal for the second year; of which, find the amount for the second year as before; and so on, through every year to the last.

2. Subtract the given principal from the last amount, and the remainder is the compound interest.

EXAMPLES.

1. What is the compound interest of L. 400 for 4 years, at 6 per cent. per annum?

Principal.....L. 400
Rate of interest. 6

24·00

	L.
Principal for the 1st. year.....	400
Interest of ditto.....	24
Principal for the 2d. year.....	424
	6
	25·44
	20
	8·80
	12
	9·60
	4
	2·40

	L.		
Principal for the 2d. year	424		
Interest for do.	25	8	$9\frac{1}{2}$
Principal for the 3d. year	449	8	$9\frac{1}{2}$

6

26·96 12 9

20

19·32

12

3·93

4

3·72

	L.	s.	d.
Principal for the 3d. year	449	8	$9\frac{1}{2}$
Interest for do.	26	19	$3\frac{3}{4}$
Principal for the 4th. year	476	8	$1\frac{1}{4}$

6

28·58 8 $7\frac{1}{2}$

20

11·68

12

8·23

	L.	s.	d.
Principal for the 4th. year	476	8	$1\frac{1}{4}$
Interest for do.	28	11	8
Amount for the 4th. year	504	19	$9\frac{1}{4}$
Subtract the first principal	400		
The compound interest, or Answer required	104	19	$9\frac{1}{4}$

Or thus,

L.		L.			
5	$\frac{1}{20}$	400			..... *Principal.*
1	$\frac{1}{5}$	20			
		4			
5	$\frac{1}{20}$	424			..... *Amount for the 1st. year.*
1	$\frac{1}{5}$	21	4		
		4	4	$9\frac{1}{2}$	
5	$\frac{1}{20}$	449	8	$9\frac{1}{2}$	*Ditto for the 2d.*
1	$\frac{1}{5}$	22	9	$5\frac{1}{4}$	
		4	9	$10\frac{1}{2}$	
5	$\frac{1}{20}$	476	8	$1\frac{1}{4}$	*Do. for the 3d.*
1	$\frac{1}{5}$	23	16	$4\frac{1}{4}$	
		4	15	$3\frac{1}{4}$	
		504	16	$9\frac{1}{4}$	*Do. for the 4th.*
		400			
		L. 104	19	$9\frac{1}{4}$	*the Answer.*

2. What is the amount of L. 240 for 5 years, at 5 per cent. per annum, compound interest?

Ans. L. 306 6 $1\frac{9}{4}d.$

3. What is the compound interest of L. 120 10*s.* for 6 years and 7 months, at 6 per cent. per annum?

Ans. L. 56 8*s.* $2\frac{1}{4}d.$

4. Required to find the several amounts of L. 60 payable yearly, half yearly, and quarterly, being forborn $3\frac{1}{2}$ years, at 5 per cent. per annum, compound interest?

Ans. L. 71 3*s.* $10\frac{1}{4}d.$, L. 71 6*s.* $4\frac{1}{4}d.$, and L. 71 7*s.* $10\frac{1}{4}d.$

COMPOUND INTEREST BY DECIMALS.

RULE.*

1. Find the amount of L. 1 for 1 year at the given rate *per cent.*
2. Involve the amount thus found to that power, whose index is denoted by the number of years.
3. Multiply this power by the principal, or given sum, and the product will be the amount.
4. Subtract the principal from the amount, and the remainder is the interest.

---

* Let $\begin{cases} r = \text{amount of L. 1 for 1 year at the given rate } \textit{per cent.} \\ p = \text{principal.} \\ i = \text{interest.} \\ t = \text{time.} \\ m = \text{amount for the time.} \end{cases}$

Then the following Theorems will shew the solutions of all the cases in compound interest.

I. $pr^t = m.$ II. $pr^t - p = i.$

III. $\sqrt[t]{\frac{m}{p}} = r.$ IV. $\sqrt[t]{\frac{m}{p}} - 1 = r.$

Or the Theorem for the time, as well as for all the other cases, may be given by logarithms, as follow:

I. $t \times log.\ r + log.\ p = log.\ m.$

II. $log.\ m - t \times log.\ r = log.\ p.$

III. $\frac{log.\ m - log.\ p}{t} = log.\ r.$

EXAMPLES.

1. What is the compound interest of L. 300 for 4 years, at 6 per cent. per annum?

1·06 = *amount of L. 1 for 1 year, at 6 per cent.*
1·06

636
106

1·1236
1·06

67416
11236

1·191016
1·06

7146096
1191016

1·26247696 = *4th. power of L. 1·06.*
300 = *principal.*

378·74308800 = *amount.*
300

78·74308800 = *L. 78 14s. 10¼d. the Ans.*

IV. $\sqrt{\frac{log.\ m - log.\ p}{log.\ r}} = t.$

If the amount of any sum be required for the parts of a year, it may be determined as follows:

First, When the time is any aliquot part of a year

RULE.

1. Find the amount of L. 1 for 1 year at the rate *per cent.*

2. What is the compound interest of 200 dollars for 4 years, at 6 per cent. per annum?

Ans. 52 D. 49 c. $5\frac{49}{53}$ m.

3. What is the amount of $375\frac{1}{2}$ dollars for 5 years, at 5 per cent. per annum, compound interest?

Ans. 479 D. 2 d. 4 c. 3 m.

4. What is the amount of 480 dollars for $6\frac{1}{2}$ years, at 6 per cent. per annum, compound interest?

Ans. 701 D. $31\frac{1}{2}$ cents.

DISCOUNT BY COMPOUND INTEREST.

RULE.*

Divide the given sum or debt by that power of the amount of L. 1 for 1 year, denoted by the time, and the

---

2. Extract that root, of this amount, whose exponent is equal to the aliquot part, and it will be the amount of L. 1 for the given time.

3. Multiply the last amount by the given principal, and the product is the amount required.

Secondly, When the time is not an aliquot part of a year.

RULE.

1. Find the amount of L. 1 for 1 year as before, and the 365th. root of it will be the amount for one day.

2. Raise this amount to the power, whose index is equal to the number of days in the given time, and it will be the amount of L. 1 for that time.

3. Multiply the last amount by the principal, and the product will be the answer required.

NOTE, To avoid extracting very high roots, the same may be performed much easier by logarithms, thus: divide the logarithm of the amount of L. 1 for 1 year by the denominator of the aliquot part, and the quotient will be the logarithm of the root sought.

* Let m=sum or debt to be discounted, and the other letters as before, then will the following Theorems express all the cases in this Rule.

quotient will be the present worth, which, subtracted from the given sum or debt, will leave the discount.

EXAMPLES.

1. What is the present worth and discount of L. 150 due 3 years hence, discounting at 5 *per cent. per annum*, compound interest?

$$\overline{1\cdot05}\Big|^{3} = 1\cdot157625) \ 150\cdot000000 \ (\ 129\cdot575 = \text{L. } 129\ 11s.\ 6d.$$

present worth, and L. 150 — L. 129 11*s.* 6*d.* = L. 20 8*s.* 6*d. discount.*

2. What is the discount of 4000 dollars, due 4 years hence, at 6 *per cent. per annum*, compound interest?

Ans. 831 *D.* 62 *c.* 6 *m.*

ANNUITIES.

An Annuity is a sum of money payable every year for a certain number of years, or forever.

When the debtor keeps the annuity beyond the time of payment, it is said to be in *arrears.*

I. $\frac{m}{r^t} = p.$ II. $pr^t = m.$

III. $\frac{m}{p} = r^t.$

which being continually divided by r, till nothing remain, the number of these divisions will be equal to t, and that root of r, r^t whose exponent is denoted by t, is equal to r.

The sum of all the annuities for the time they have been forborn, together with the interest due upon each, is called the *amount*.

If an annuity be to be bought, or paid all at once at the beginning of the first year, the price, which ought to be given for it, is called the *present worth*.

To find the Amount of an Annuity at Simple Interest.

RULE.*

1. Find the sum of the series 1, 2, 3, 4, &c. to the number of years less one.

2. Multiply one year's interest of the annuity by this sum, and the product will be the interest due upon the annuity.

3. To this product add the rectangle of the annuity and time, and the sum will be the amount required.

---

* Let n be the annuity, r the ratio, t the time, and a the amount.

Then will the following Theorems give the solutions of all the different cases.

I. $\dfrac{nrt^2-nrt}{2}+nt=a$

II. $\dfrac{2a}{t^2r+2t-rt}=n.$

III. $\dfrac{2a-2tn}{nt^2-nt}=r.$

IV. $\left(\dfrac{2a}{nr}+\dfrac{d}{4}\right)^{\frac{1}{2}}-\dfrac{d}{2}=t.$

In the last Theorem $d=\dfrac{2n-nr}{nr}$

NOTE, When the annuity is to be paid half yearly or quarterly; then take, in the former case, half of the annuity, half of the ratio, and double the number of years; but, in the latter case, take one fourth of the annuity, one fourth of the ratio, and four times the number of years, and proceed by the rule.

EXAMPLES.

1. What is the amount of an annuity of L. 60 for 6 years, allowing simple interest, at .6 *per cent. per annum*?

1+2+3+4+5=15=*Sum of the number of years less one.*

L. 3 12*s.*=1 *year's interest of the annuity.*

5

18 0

3

54 0 = *whole interest due.*

Add 360 0 = 60×6=*annuity multiplied by the time*

L. 414 0 = *amount required.*

2. If a salary of 500 dollars remain unpaid 8 years; what is the amount, at 6 *per cent. per annum*, simple interest? Ans. 4840 *D.*

3. What will an annuity of 800 dollars amount to in 10 years, at 5 *per cent. per annum*?

Ans. 9800 *Dollars.*

4. What is the amount of an annuity of 600 dollars for 9 years, to be paid by half yearly payments, at 6 *per cent. per annum*? Ans. 6777 *D.*

To find the present Worth of an Annuity at Simple Interest.

RULE.*

Find the present worth of each year by itself, discounting from the time it becomes due, and the sum of these will be the present worth required.

---

* The purchasing of annuities at simple interest is in the highest degree unjust and absurd. The truth of this assertion will evidently appear in this single example, *viz.* The price of an annuity of L. 30 to continue 40 years discounting at 6 *per cent. per annum*, will, by either of the above Rules, amount to a sum of which one year's interest only will exceed the annuity.

Would it not, therefore, be highly inconsistent to give, for an annuity to continue only 40 years, a sum of money which would yield a greater interest forever?

The estimation of annuities at simple interest is given chiefly for speculation.

The most equitable way of purchasing annuities is at compound interest.

Let p=present worth, and the other letters as before.

Then
$$n \times \frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r}, \&c.\ to \frac{1}{1+tr} = p.$$
$$p \div \frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r}, \&c.\ to \frac{1}{1+tr} = n.$$

The other two Theorems for the rate and time cannot be given in general terms.

Examples.

1. What is the present worth of an annuity of L. 40 to continue 5 years, discounting at 6 *per cent. per annum*, simple interest?

L.	*L.*	*L.*	*L.*	
106	100	40	37·7358	=*present worth for* 1 *year.*
112	100	40	35·7142	=.....2 years.
118	100	40	33·8983	=.....3 years.
124	100	40	32·2580	=.....4 years.
130	100	40	30·7692	=.....5 years.

170·3755=L. 170 7*s.* $6\frac{3}{7}$*d.*=*present worth of the annuity required.*

2. What is the present worth of an annuity or pension of 500 dollars to continue 6 years, at 6 *per cent. per annum*, simple interest? Ans. 2497 *D.* 34 *c.* 1 *m.*

To find the Amount of an Annuity at Compound Interest.

Rule.*

---

* Let r=rate or amount of L. 1 for 1 year, and the other letters as before.

Then will the following Theorems exhibit the solutions of all the cases.

I. $\frac{r^t-1}{r-1}\times n=a.$ II. $\frac{ar-a}{r^t-1}=n.$

III. $\sqrt[t]{\frac{ar+n-a}{n}}=r.$

1. Make 1 the first term of a geometrical progression, and the amount of L. 1 for 1 year at the given rate *per cent.* the ratio.

2. Carry the series to as many terms as the number of years, and find its sum.

3. Multiply this sum by the given annuity, and the product will be the amount required.

Examples.

1. What is the amount of an annuity of L. 60 for 4 years, allowing compound interest, at 6 *per cent. per annum* ?

$$1+1\cdot06+\overline{1\cdot06}\,\vert^{2}+\overline{1\cdot06}\,\vert^{3} = 4\cdot374616.$$

$4\cdot374616$ = *Sum.*
60 = *Annuity.*

$262\cdot476960$
20

$9\cdot539200$
12

$6\cdot470400$
4

$1\cdot881600$

Ans. L. 262 9*s*. $6\frac{1}{4}$*d*.

2. Find the amount of an annuity of 80 dollars for 5 years, at 5 *per cent. per annum*, compound interest.

Ans. 442 *D*. 5 *c*. $0\frac{1}{2}$*m*.

Which being continually divided by r till o remain, the number of divisions will be equal to t. Or, that root of it, whose exponent is equal to t, will be equal to r.

3. If 120 dollars yearly rent, or annuity, be forborn 8 years; what will it amount to, at 6 *per cent. per annum*, compound interest?

Ans. 1187 *D.* 69 *c.* 6*m.*

To find the present Worth of Annuities at Compound Interest.

RULE.*

1. Divide the annuity by the ratio, or amount of L. 1 for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth of the annuity for two years.

---

* Let p=present worth of the annuity, and the other letters as before.

Then the following Theorems, where the purchase of annuities is concerned, will express the solutions of all the cases.

$$\text{I.}\quad a \times \frac{r^t - 1}{r^{t+1} - r^t} = p.$$

$$\text{II.}\quad p \times \frac{r^{t+1} - r^t}{r^t - 1} = a.$$

$$\text{III.}\quad \frac{a}{p + a - pr} = r^t.$$

which being divided by r till o remain, the number of those divisions will be equal to t, or the root of r^t, whose index is equal to t, will be equal to r.

3. Find, in like manner, the present worth of each year by itself, and the sum of all these is the answer.

Examples.

1. What is the present worth of an annuity of L. 50 to continue 4 years, discounting at 6 *per cent. per annum*, compound interest?

Ratio = 1·06)50·000000 (47·1698=*present worth for* 1 *year.*

$\overline{1\cdot06}|^{2}$ = 1·1236)50·0000000(44·4998= . . . . . 2 *years.*

$\overline{1\cdot06}|^{3}$ = 1·191016)50·0000000(41·9809= . . . . . 3 *years.*

$\overline{1\cdot06}|^{4}$ = 1·26247696)50·0000000(39·6046= . . . . . 4 *years.*

173·2551 = L. 173 5s. 1d.

=*whole present worth of the annuity required.*

2. What is 40 dollars *per annum* to continue 20 years, worth in present money, at 6 *per cent. per annum*, compound interest? Ans. 458 *D.* 79 *c.* $6\frac{4}{5}$ *m.*

To find the present Worth of a Freehold Estate or Annuity to continue forever, at Compound Interest.

Rule.*

---

* The following Theorems exhibit all the varieties in this Rule.

I. $\frac{n}{r-1}=p.$ II. $\overline{r-1\times p}=n.$

III. $\frac{n}{p}+1=r.$

As the rate *per cent.* is to L. 100; so is the annuity, or yearly rent to the value required.

Examples.

1. An estate brings in annually L. 80; what would it sell for, allowing the purchaser 4 *per cent.* compound interest for his money?

L.	*L.*	*L.*
4 :	100 : :	80 :
	80	
	4)8000	
	L. 2000	*the Ans.*

2. What is the price of a perpetual annuity of 160 dollars, discounting at 5 *per cent.* compound interest?

Ans. 3200 *dollars.*

3. Required to find the value of a freehold estate, which brings in 200 dollars a year, allowing the buyer 6 *per cent.* for his money?

Ans. $3333\frac{1}{3}$ *dollars.*

To find the present Worth of an Annuity, or Freehold Estate in Reversion, at Compound Interest.

The price of a freehold estate, or annuity to continue forever, at simple interest, would be expressed by

$$\frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r} + \frac{1}{1+4r}, \text{ \&c. } \textit{without end};$$

but the sum of this series is infinite, which shews the absurdity of using simple interest in these cases.

RULE.*

1. Find the present worth of the annuity as though it were to be entered on immediately.

2. Find the present worth of the last present worth, discounting for the time between the purchase and the commencement of the annuity, and it will be the answer required.

EXAMPLES.

1. Suppose a freehold estate of L. 30 *per annum* to commence 3 years hence, were to be sold; what is its value, allowing the purchaser 5 *per cent.* ?

L.	*L.*	*L.*
5	: 100	:: 30
	30	
	6)3000	

L. 600 = *present worth if entered on immediately.*

and $1{\cdot}05^{3} = 1{\cdot}157625)600{\cdot}000000(518{\cdot}3025 = \text{L.}518$ 6*s.* $0\frac{1}{4}d.$ = *present worth of* L. 600 *for* 3 *years, or the whole present worth required.*

2. The reversion of a freehold estate of 120 dollars *per annum* to commence 4 years hence, is to be sold;

---

* The following Theorems shew all the cases under this Rule.

I. $\frac{n}{r-1} = p$; then change p into m, and $\frac{\frac{m}{v}}{r} = p$.

II. $pr^{v} = m$; then change m into p, and $\frac{prr-pr}{r} = n$.

what is its worth in ready money, allowing the purchaser 6 *per cent.* ? Ans. 1584 *D.* 18 *c.* 7 *m.*

3. Which is the most advantageous, and how much, a term of 8 years in an estate of 240 dollars *per annum*, or the reversion of such an estate forever, after the expiration of the said 8 years, calculating at the rate of 6 *per cent.* compound interest ?

Ans. the reversion is more advantageous than the first term of 8 years by 1019 *D.* 30 *c.*

POSITION.

Position is a rule, by which, from false or supposed numbers taken at pleasure, the true ones are discovered, and is distributed into two kinds ; single and double.

SINGLE POSITION.

Single Position is that, which teaches to resolve those questions, whose results are proportional to their suppositions.

Rule.

1. Take any number, and perform the same operations with it, as are described to be performed in the question.
2. Then say, as the result of the operation is to the result in the question ; so is the position to the answer required.

Proof.

Add the several parts of the answer together, and if the sum be equal to the result in the question, the work is right.

Examples.

1. A, B, and C share L. 240 among them; A's share is double to that of B's, and B's share is triple of C's; what is the share of each?

Suppose A's share to be L. 90
Then will B's $= \frac{90}{2} = 45$
And C's $= \frac{45}{3} = 15$

L. 150 *Sum.*

L. L. L.
As 150 : 240 :: 90 :
90

15|0)2160|0(144=*A's share.*
15
66
60
60
60
..

Consequently $\frac{144}{2} = 72 =$ B's.
And $\frac{72}{3} = 24 =$ C's.

L. 240 *Proof.*

2. A School-master being asked how many scholars he had, replied, if I had as many more as I now have, half as many, one fourth and one eighth as many, I should then have 115; how many scholars had he?

Ans. 40.

3. A person, after spending $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$ of his money, had L. 52 left; how much had he at first?

Ans. L. 240.

4. A certain sum of money is to be divided between 4 men, in such a manner, that A shall have $\frac{1}{2}$, B $\frac{1}{4}$, C $\frac{1}{7}$, and D the remainder, which is L. 20; what is the sum?

Ans. L. 400.

5. A and B discoursing concerning their ages; B said his age was twice the age of A; C said his age was twice the age of both, and that the sum of their ages was 90; what was the age of each?

Ans. *A's* 10, *B's* 20, *and C's* 60.

6. What number is that, which being increased by its $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$, the sum will be $113\frac{1}{14}$.

Ans. 60.

DOUBLE POSITION.

Double Position is that, which teaches to resolve questions, in which the results are not proportional to their positions, by making two suppositions of false numbers.

Rule.

1. Take any two convenient numbers, and proceed with each, according to the conditions of the question.

2. Find how much the results are different from the result in the question.

3. Multiply each errour by the contrary supposition, and find the sum and difference of the products.

4. If the errours be a like, divide the difference of the products by the difference of the errours, and the quotient will be the answer.

5. If the errours be unlike, divide the sum of the products by the sum of the errours, and the quotient is the answer.

Note 1, The errours are said to be alike, when they

are both too great or too little; and unlike, when one is too great and the other too little.

NOTE 2. When the errours are the same in quantity, but unlike in quality, half the sum of the suppositions is the number required.

EXAMPLES.

1. A lady bought lustring at 10*s.* a yard, and linen at 3*s.* a yard; the number of yards of lustring and linen was 16, and the whole cost L. 4 10*s.*; how many yards had she of each?

Suppose 8 *yards of lustring, value* 80*s.*
Then she must have 8 *yards of linen, value* 24

Sum of their values 104

So that the first errour is + 14

Again, suppose she had 7 *yards of lustring, at* 70*s.*
Then she must have 9 *yards of linen, at* 27

Sum of their values 97

So that the second errour is + 7

Then 14—7=7=*difference of the errours.*
Also 8×7=56=*product of the first supposition and second errour.*
And 7×14=98=*product of the second supposition by the first errour.*
And 98—56=42=*difference of the products.*
Whence 42÷7 =6 =*yards of lustring.* } *the Ans.*
And 16—6 =10=*yards of linen.* }

2. Two persons, A and B, laid out equal sums of money in trade, A gained 200 dollars and B lost 200 dollars, so that A's money is now triple of B's; what did each lay out? Ans. 400 *dollars.*

3. A labourer was hired for 50 days upon this condition, that for every day he wrought he should receive 60 cents, and for every day he was idle, should forfeit 20 cents; at the expiration of the time, he received in all 10 dollars; how many days did he work, and how many was he idle.

Ans. *He wrought* 25 *days, and was idle* 25.

4. There is a fish, whose head is 6 feet long; his tail is as long as his head and half the length of his body, and his body is as long as his head and tail; what is the whole length of the fish? Ans. 48 *feet.*

5. Two persons, A and B, have both the same income; A saves $\frac{1}{7}$ of his yearly; but B, by spending 200 dollars per annum more than A, at the end of 6 years, finds himself 1000 dollars in debt; what is their yearly income, and what do they spend *per annum?*

Ans. Their yearly income is $166\frac{2}{3}$ *dollars, also, A spends* $133\frac{1}{3}$ *dollars, and B* $333\frac{1}{3}$ *dollars per annum.*

6. Four merchants, A, B, C, and D, built a ship, which cost them 10000 dollars, of which A paid a certain sum, B paid 500 dollars more than A, C paid 500 dollars more than B, and D paid twice as much as A, B, and C; they fixed her for sea with a cargo worth $\frac{3}{4}$ of $\frac{4}{5}$ of the ship; the outsits and charges of the voyage amounted to $\frac{1}{7}$ of $\frac{3}{8}$ of her cargo; upon her return, they found their clear gain to be $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of the ship, cargo, and expenses: please to inform me what the ship cost them, severally; what share each had in her, and what, upon the final adjustment of their accompts, they had severally gained?

Ans. A owned $\frac{11}{180}$ *of the ship, which cost him* $611\frac{1}{9}$ *dollars, and his share of the gain was* $204\frac{1}{8}\frac{3}{8}$ *dollars; B owned* $\frac{1}{9}$ *of the ship, which cost him* $1111\frac{1}{9}$ *dollars, and his gain was* $372\frac{6}{9}$ *dollars; C owned* $\frac{29}{180}$ *of the ship, which cost him* $1611\frac{1}{9}$ *dollars, and his gain was* $539\frac{1}{1}\frac{3}{8}$ *dollars, and D owned* $\frac{2}{5}$ *of the ship, which cost him* $6666\frac{2}{3}$ *dollars, and his gain was* $2233\frac{1}{3}$ *dollars.*

PERMUTATION AND COMBINATION.

The Permutation of Quantities is the shewing how many different ways any given number of things may be changed.

This is also called Variation, Alternation, or changes; and the only thing to be regarded here is the order in which they stand; for no two parcels are to have all their quantities placed in the same situation.

The *Combination of Quantities* is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order in which they stand.

This is sometimes called *Election or Choice;* and here every parcel must be different from all the rest, and no two are to have precisely the same quantities or things.

The *Composition* of *Quantities* is the taking of a given number of quantities out of so many equal rows of different quantities, one out of each row, and combining them together.

Here no regard is had to their places; and it differs from combination only, as that admits of but one row, or set of things.

Combinations of the same form are those, in which there are the same number of quantities, and the same repetitions thus: *aabb*, *ccff*, *eedd*, &c. are of the same form; but *abce*, *bdee*, *aaab*, &c. are of different forms.

PROBLEM I.

To find the number of permutations that can be made of any given number of things, all different from one another.

RULE.

Multiply continually together all the terms of the natural series, from 1 up to the given number, and the last product will be the answer required.

EXAMPLES.

1. How many changes may be made with the letters *abcd*?

1
2
—
2
3
—
6
4
—
24 *the Ans.*

2. How many days can 6 persons be placed in a different position at dinner ?

Ans. 720.

3. How many changes may be rung on 10 bells, and what time would it require, supposing 10 changes to be rung in 1 minute ?

Ans. 3625800 changes, and 251 days, 19 hours.

PROBLEM 2.

Any number of different things being given, to find how many changes can be made out of them, by taking any given number at a time.

RULE.

Take a series of numbers, beginning at the number of things given, and decreasing by 1 to the number of things to be taken at a time, and the product of all the terms will be the answer.

EXAMPLES.

1. How many changes may be made out of the 4 letters, *abcd*, by taking 3 at a time ?

4
3
—
12
2
—
24

Or, $4 \times 3 \times 2 = 24$ *the Answer.*

2. How many words can be made with 6 letters of the alphabet, admitting that a number of consonants may make a word ? Ans. 165765600.

PROBLEM 3.

Any number of things being given, of which there are several given things of one sort, several of another, &c. to find how many changes can be made out of them all.

RULE.

1. Take the series 1, 2, 3, 4, &c. up to the number of things given, and find the product of all the terms.

2. Take the series 1, 2, 3, 4, &c. up to the number of the given things of the first sort, and the series 1, 2, 3, 4, &c. up to the number of the given things of the second sort, &c.

3. Divide the product of all the terms of the first series by the joint product of all the terms of the remaining series, and the quotient is the answer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Netothathite?*

$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12$ (= *number of letters in the word*) = 479001600.

1×2 (= *number of e's*) = 2.
1×2 (= *number of h's*) = 2.
$1\times2\times3\times4$ (= *number of t's*) = 24.
$24\times2\times2=96$)479001600(4989600 *the Ans.*
384

950
864

860
768

921
864

576
576

2. What is the variety in the succession of the following musical notes, *fa, fa, sol, sol, la, la, mi, fa*?
Ans. 3360.

PROBLEM 4.

To find the changes of any given number of things, taken a given number at a time; in which there are several given things of one sort, several of another, &c.

Rule.

1. Find all the different forms of combination of all the given things, taken as many at a time as are required in the question.

2. Find the number of changes in any form, by Problem 3d. and multiply it by the number of combinations in that form.

3. Do the same for every distinct form; and the sum of all the products will be the number of changes required.

Examples.

1. How many changes can be made of every 4 letters out of these 8, *aabbbccc*?

No. of forms.	*forms.*	*No. of combinations.*	*No. of changes in each form.*
1*st.*	b^3a, b^3c, c^3a, c^3b	4	$\left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2\times3 \quad = 6. \end{array}\right.$ —=4.
2*d.*	a^2b^2, a^2c^2, b^2c^2;	3	$\left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2\times1\times2= 4 \end{array}\right.$ —=6.
3*d.*	a^2bc, b^2ac, c^2ab, ...	3	$\left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2 \quad = 2 \end{array}\right.$ —=12.

Therefore $\left\{\begin{array}{l} 4\times4=16 \\ 6\times3=18 \\ 12\times3=36 \end{array}\right.$

70= *number of changes required.*

2. How many alternations can be made of every eight figures out of these ten; 4444332261?

Ans. 22260.

PROBLEM 5.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE.

1. Take the series 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.
2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.
3. Divide the last product by the former, and the quotient will be the number sought.

EXAMPLES.

1. How many combinations can be made of 8 letters out of 12?

$1\times2\times3\times4\times5\times6\times7\times8$(=*number to be taken at a time*)$\times40320$.

$12\times11\times10\times9\times8\times7\times6\times5$(=*Same number from* 12)$=19958400$.

4032,0)1995840,0(495 *the Ans.*
16128

38304
36288

20160
20160

.....

2. How many combinations can be made of 4 letters out of the 26 letters of the alphabet?

Ans. 14950.

PROBLEM 6.

To find the number of combinations of any given number of things, by taking any given number at a time; in which there are several things of one sort, several of another, &c.

RULE.

1. Find, by trial, the number of different forms, of which the things, to be taken at a time, will admit, and the number of combinations in each.

2. Add together the number of combinations, thus found, and the sum will be the number required.

EXAMPLES.

1. Let the things proposed be *aaabbbccd*; it is required to find the number of combinations which may be made of every 4 of these quantities.

	Terms.	*Combinations.*
1.	a^3b, a^3c, a^3d, b^3a, b^3c, b^3d,	6
2.	a^2b^2, a^2c^2, b^2c^2,	3
3.	a^2bc, a^2bd, a^2cd, b^2ac, b^2ad, b^2cd, c^2ab, c^3ad, c^2bd,	9
4.	$abcd$,	1
		19 = *number of combinations required.*

2. How many combinations are there in *aaabbbccde*, taking 5 at a time? Ans. 13.

PROBLEM 7.

To find the compositions of any number, in an equal number of sets, the things themselves being all different.

RULE.

Multiply the number of things in every set continually together, and the product will be the answer required.

EXAMPLES.

1. Suppose there are 4 companies, in each of which there are 8 men; how many ways may 4 men be chosen, 1 out of each company?

8
8
—
64
8
—
512
8
—
4096 *the Ans. Or*, 8×8×8×8=4096.

2. How many chances are there in throwing 6 dice?
Ans. 46656.

MISCELLANEOUS QUESTIONS.

1. What part of 8*d.* is $\frac{3}{5}$ of 5*d.*? Ans. $\frac{5}{14}$.
2. What number is that, from which $\frac{5}{8}$ being taken, the remainder will be $\frac{1}{3}$? Ans. $2\frac{1}{24}$.
3. What number is that, to which if $\frac{1}{2}$ of $\frac{1}{4}$ of 4 be added, the sum will be 8? Ans. $6\frac{1}{2}$.

4. There is a pole, $\frac{1}{10}$ of its length stands in the ground, 8 feet of it in the water, and $\frac{7}{8}$ of it in the air; what is the whole length of the pole? Ans. 320 *feet.*

5. What number is that, of which $18\frac{4}{13}$ is $\frac{4}{7}$ of it?
Ans. $32\frac{1}{10}$.

6. Two ships sailed from a certain port at the same time, one sailed due north 60 leagues in 2 days, and the other 45 leagues due east in the same time; how far were they assunder the second day? Ans. 75 *leag.*

7. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{5}$ plumbs, 40 of them peaches, and 30 cherries; how many trees does the orchard contain?
Ans. 840.

8. Divide 400 Eagles among 20 men; give A 4 dimes more than B, B 4 dimes more than C, &c.
Ans. T's share=196 D. 20 c.

9. Bought 40 yards of broad cloth for four times as many dollars, and sold it again for five times as much; but if it had cost me as much as I sold it for; what should I have sold it for, to gain after the same rate?
Ans. 250 D.

10. The account of a certain school is as follow; *viz.* $\frac{1}{10}$ of the school study geometry, $\frac{1}{8}$ learn grammar, $\frac{1}{2}$ learn arithmetic, $\frac{1}{4}$ learn to write, and 5 learn to read; what is the number of each?

Ans. 5 geometers, 10 grammarians, 40 arithmeticians, 20 writers, and 5 readers.

11. A and B together can do a piece of work in 40 days; with the assistance of C, they can do it in 24 days; in what time would C do it by himself?
Ans. 60 days.

12. A gentleman left his son a fortune; $\frac{9}{17}$ of which he spent in 8 months; $\frac{1}{3}$ of $\frac{5}{8}$ of the remainder lasted him 4 months longer, then he had only 800 dollars left; pray, what did his Father bequeath him?
Ans. 2720 dollars.

13. There is an Island, 60 miles in circumference, and 3 men set out together to travel the same way about it; A goes 8 miles a day, B 9, and C 10; when will they all come together again, and how far will each travel?

Ans. They will all come together again in 60 days; A will travel 480 miles, B, 540, and C, 600.

14. Three men are employed to perform a piece of work for L. 2 10s; A and B are supposed to do $\frac{3}{17}$ of the work, A and C $\frac{5}{17}$, and B and C $\frac{4}{17}$ of it; they are to be paid proportionally; can you divide it as it should be?

Ans. A's proportion is 19s. 8 $1\frac{4}{5}$qr. B's. 9s. 2d. 2 $\frac{2}{5}$qr. and C's. L. 1 1s. 0d. $3\frac{1}{5}$qr.

15. Twenty Generals, 30 Colonels, 24 Majors, and 24 Captains spent at a dinner 320 dollars, which sum was divided among them in such a manner, that 4 Generals paid as much as 5 Colonels, 10 Colonels as much as 16 Majors, and 8 Majors as much as 12 Captains; the question is to know the sum of money paid by all the Generals, also by the Colonels, Majors, and Captains.

Ans. The 20 Generals paid 100 dollars, the 30 Colonels 120 D. the 24 Majors 60 D. and the 24 Captains 40 D.

16. Required the least vulgar fraction equivalent to ·714285.

Ans. $\frac{5}{7}$.

17. A can do a piece of work in 6 days; B can do twice as much in 8 days, and C thrice as much in 9 days; in what time can they finish it jointly?

Ans. $1\frac{1}{3}$ day.

18. A general, disposing his army into a square, found he had 360 soldiers over and above; but increasing each side with one soldier, he wanted 41 to fill up the square: how many soldiers had he?

Ans. 40360.

19. Three men purchased a ship in company, toward the payment of which, A advanced $\frac{1}{3}$, B $\frac{4}{7}$, and C 1500 dollars; what did A and B pay, and what part of the ship had C?

Ans. A paid 1312$\frac{1}{2}$ dollars, B 3750 dollars, and C's part of the ship was $\frac{8}{21}$.

20. A, B, C, and D companied, and gained a sum of money, of which A, B, and C took 300 dollars; B, C, and D, 400 dollars; C, D, and A, 380 dollars, and D, A, and B, 360 dollars; what was the gain of each?

Ans. A's gain was 80 dollars, B's, 100 dollars, C's, 120 dollars, and D's, 180 dollars.

21. A person, looking on his watch, was asked the time of day, said, that it was between 3 and 4; but a more particular answer being requested, he replied, that the hour and minute hands were then together; what was the time?

Ans. 16 $\frac{4}{11}$ minutes past 3.

22. A, B, and C are to share L. 240 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively; but C dying, it is required to divide the whole sum properly between the other two.

Ans. A's L. 137 2 10 $\frac{3}{7}$d. and B's L. 102 17 1$\frac{5}{7}$d.

23. Reduce $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ of $\frac{4}{5}$ to equivalent fractions, having the least common denominator.

Ans. $\frac{10}{20}$, $\frac{5}{20}$, and $\frac{12}{20}$.

24. I sold a watch for L. 30, and by so doing, lost 15 per cent. whereas I ought to have cleared 20 per cent.; how much was it sold under its value? Ans. L. 12 7$\frac{3}{17}$s.

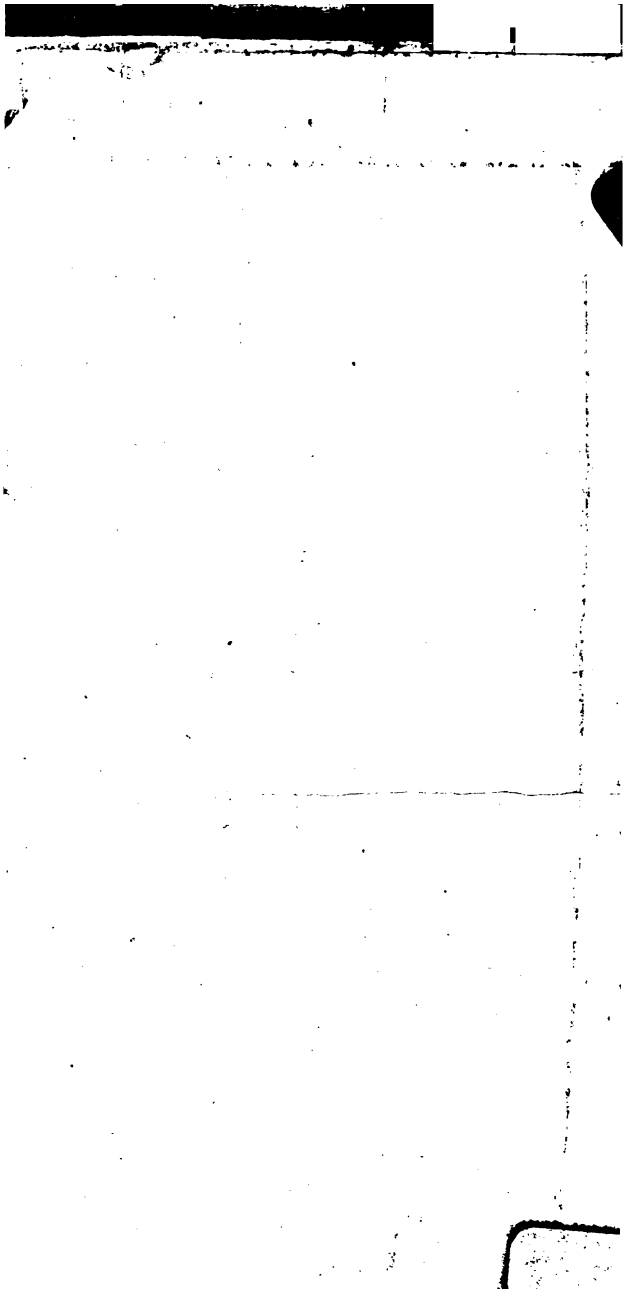
25. What is the square root of 150$\frac{1}{16}$? Ans. 12$\frac{1}{4}$.

26. A person, who was possessed of $\frac{3}{8}$ of a ship, sold $\frac{2}{3}$ of his share for 1840 dollars; what was the value of the ship at that rate? Ans. 7360 dollars.

27. A young hare starts 86 yards before a grey hound, and is not perceived by him till she has been running one minute; she scuds away at the rate of 12 miles an hour, and the dog, on view, makes after her at the rate of 20 miles an hour; how long will the course hold, and what length of ground will be run over from the out-sitting of the dog? Ans. 1 min. $51\frac{8}{11}$ sec. and 1095 yards.

28. A gentleman being determined to pay L. 13 among his labourers for a days work, would give to every man 3*s.* to every woman 2*s.* and to every boy 1*s.* 6*d.*; the number of men, women, and boys was the same; what was the number of each? Ans. 40.

29. A gentleman divided his fortune among his children; to A he gave L. 8 as often as to B L. 5; and to C, but L. 3 as often as to B L. 7, and yet C's portion was L. 4000; what was the whole legacy?
Ans. L. 28266 13*s.* 4*d.*

30. A water tub holds 200 gallons; the pipe, which conveys the water to it, usually admits 10 gallons in 7 minutes, and the tap discharges 13 gallons in 12 minutes; now, supposing these to be carelessly left open, and the water begin to run at 3 o'clock in the morning; a servant, at 5, finding the water running, shuts the tap, and is solicitous to know in what time, after this accident, the tub will be full.
Ans. 51 minutes after 6.

THE END.